A short update on the Beam Energy measurement from PI Test RFQ

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Outline

• **Mistakes done in beam energy error measurements**
  – Using LINEST function in EXCEL
  – Standard Statistical calculations (Bevington)

• **Possible errors in the measurements**

• **Views on using fixed BPMs for energy measurements**

• **Summary**
Layout of the PI-Test MEBT 1.1
Energy measurement data taken on 2\textsuperscript{nd} June, 2016 using Movable BPM (data taken by me & Jonathan)

Least Square fit

$$a = \frac{1}{\Delta} \left( \frac{\sum \frac{y_i}{\sigma_i^2} \cdot \sum \frac{x_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \cdot \sum \frac{y_i}{\sigma_i^2}}{\sum \frac{1}{\sigma_i^2} \cdot \sum \frac{y_i}{\sigma_i^2}} \right)$$

$$b = \frac{1}{\Delta} \left( \frac{\sum \frac{1}{\sigma_i^2} \cdot \sum \frac{x_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \cdot \sum \frac{y_i}{\sigma_i^2}}{\sum \frac{1}{\sigma_i^2} \cdot \sum \frac{x_i}{\sigma_i^2}} \right)$$

$$\Delta = \frac{\sum \frac{1}{\sigma_i^2} \cdot \sum \frac{y_i}{\sigma_i^2}}{\sum \frac{1}{\sigma_i^2} \cdot \sum \frac{x_i}{\sigma_i^2} - \left( \sum \frac{x_i}{\sigma_i^2} \right)^2}$$

$$\sigma_a^2 = \frac{\sum \frac{1}{\sigma_i^2} \cdot \sum \frac{x_i}{\sigma_i^2} \left( \sum \frac{1}{\sigma_i^2} \right)^2 - \frac{2}{\sigma_i^2} \sum \frac{x_i}{\sigma_i^2} \sum \frac{y_i}{\sigma_i^2} + \sum \frac{1}{\sigma_i^2} \left( \sum \frac{y_i}{\sigma_i^2} \right)^2}{\sum \frac{1}{\sigma_i^2} \cdot \sum \frac{x_i}{\sigma_i^2} - \left( \sum \frac{x_i}{\sigma_i^2} \right)^2}$$

Errors in the slope:

Linest function: $2.9031 \pm 0.006$ deg/mm

Statistical analysis: $2.9047 \pm 0.038$ deg/mm

$$E(\text{MeV}) = \left[ \frac{\text{slope}}{\sqrt{\text{slope}^2 - 0.195^2}} - 1 \right] \cdot m_0c^2$$

2.126$\pm$0.009 (0.43%) MeV

2.12$\pm$0.06 (2.6%) MeV
We plotted the distribution of phase:
Looks Gaussian

The analysis showed the error bar is above the specs
Then we used the buncher on and at -90 deg and BC on
Standard deviation in phase reduced by switching on the buncher and at -90 deg phase (Bunching phase) and BC on this reduced the energy error.

We have taken the TOF BPM measurements keeping the buncher phase at -90 & BC on

After this we requested Marsh to give the raw data file in his energy measurement program
Energy of the beam using TOF BPM (30 averages & buncher at -90 deg)

Data taken by me and Jean-paul

The error in the energy measurements was within 1%
Energy error as a function of buncher phase (# of averages 30):

Energy error decreases when we are going towards bunching phase (-90 deg) and increases as we go away from -90 deg
Possible Errors in energy measurement:

- Phase of the BPM
- Mechanical (motion of BPM)
- Resonant frequency offset (effect the accurate value of energy)

\[
E = \left[ \frac{s}{\sqrt{s^2 - \left( \frac{0.36 f}{300} \right)^2}} - 1 \right] m_0 c^2
\]

\[
dE = \sqrt{\left( \frac{\partial E}{\partial f} \Delta f \right)^2 + \left( \frac{\partial E}{\partial s} \Delta s \right)^2}
\]

- Energy variation by an offset of resonant frequency by 30 kHz is 0.04%
- Energy error change by 0.05% for frequency error of 20 kHz
- So the slope error is the major contributor for the Energy error
Energy of beam using fixed BPMs:

- The energy of the beam is calculated by
  \[ \beta = \frac{s}{c(n\tau + \Delta t)}, \quad \Delta t = \frac{\Delta \phi}{0.36f} \]

- The error in the velocity is given by
  \[ \sigma_{\beta} = \sqrt{\left(\frac{\partial \beta}{\partial s}\sigma_s\right)^2 + \left(\frac{\partial \beta}{\partial f}\sigma_f\right)^2 + \left(\frac{\partial \beta}{\partial \Delta t}\sigma_{\Delta t}\right)^2} \]

- Energy errors are calculated using above formula.
- Assuming 20 kHz variation in f, and 1 mm in length.
- We observed the \( \sigma \) in phase was minimum at around bunching phase
- Absolute value of energy depends on relative phases of BPMs.
- We have observed a relative phases of BPMs have changed
Summary:

• Energy of the beam was measured with movable BPM method
  Errors are functions of buncher phase
  Minimum at -90 deg ~ 1% (major contribution from slope)

• Final RFQ energy is 2.11 ± 1% (movable BPM), which is within the specs

• Energy measurement using two fixed BPMs are also done
  Errors are less when compared to movable BPM method
  But not so reliable for absolute energy measurement

• Precession was improved using BC in the two fixed BPMs method to around 0.4%

➢ It is clearly seen that stdev of BPMs reduce with BC on

➢ Errors in energy are 0.35% with BC on and averages of 30
Thank you ...