Main PIP-II parameters

Linac

Parameters of linac beam at exit

Linac current: $I_{linac} := 2 \cdot 10^{-3}$ A

Linac Energy: $E_{linac} := 800 \cdot 10^{6}$ eV

Hor. rms norm. emittance: $\varepsilon_{nxL} := 0.30 \cdot 10^{-4}$ cm

Repetition rate: $f_{rep} := 20$ Hz

Vert rms norm. emittance: $\varepsilon_{nyL} := 0.30 \cdot 10^{-4}$ cm

Long rms norm. emittance: $\varepsilon_{nsL} := 0.35 \cdot 10^{-4}$ cm

$\frac{\varepsilon_{nsL} \cdot m_p}{c} = 1.095 \times 10^{-6}$ eV s

Beam pulse duration (copied from Booster section): $T_{inj} := 0.5369 \cdot 10^{-3}$ s

$T_{inj} \cdot f_{rep} = 1.074 \cdot \%$

$P_{linac} := I_{linac} \cdot E_{linac} \cdot T_{inj} \cdot f_{rep} \cdot 10^{-3} = 17.181$ kW

Parameters of Linac Cavities

ii :=

	0
0	"Name"
1	"f [MHz]"
2	"N cell"
3	"beta-opt"
4	"beta-geom"
5	"apert. [mm]"
6	"U_max [MV]"
7	"E.peak [MV/m]"
8	"Bpeak [mT]"
9	"G [Ohm]"
10	"R/Q [Ohm]"
11	"Q0"
12	"cavities/crm"
13	"Number of cryomodules"
14	"Stat. load @2K[W/crm]"
15	"Max. microphbics detune [Hz]"
16	"Power transfer efficiency"
17	"power margin"
18	"Accelerating phase [deg]"
19	"Static +dynamic load @ 5 K [W/crm]"
20	"Static +dynamic load @70 K [W/crm]"
21	"LFD coefficient [Hz/(MV/m)^2]"
22	"Dynamic load coupler @5K,60kW [W]"
23	"Dynamic load coupler@70K,60kW [W]"
24	...

=

	0	1	2	3	4	5
0	"MEBT"	"HWR"	"SSR1"	"SSR2"	"LB650"	"HB650"
1	162.5	162.5	325	325	650	650
2	2	2	2	2	5	5
3	0.067	0.112	0.222	0.475	0.65	0.971
4	0.56	0.094	0.186	0.398	0.61	0.92
5	20	33	30	40	88	118
6	0.1	2	2.05	4.987	11.9	19.9
7	6	44.938	38.4	40	40.6	38.9
8	0	48.321	58.1	64.5	75.3	73.1
9	0	48	84	115	193	260
10	503	272	242	297	341	610
11	$1.053 \cdot 10^{4}$	$5 \cdot 10^{9}$	$6 \cdot 10^{9}$	$8 \cdot 10^{9}$	$2.15 \cdot 10^{10}$	$3 \cdot 10^{10}$
12	1	8	8	5	3	6
13	3	1	2	7	11	4
14	0	37	13	9	2	4
15	0	20	20	20	20	20
16	0.9	0.9	0.9	0.9	0.94	0.94
17	0.8	0.8	0.8	0.8	0.8	0.8
18	90	0	0	0	0	0
19	0	60	80	62	16	32
20	0	250	255	126	48	86
21	0	0	5	2.8	1.42	1
22	0	0	0	0	0	0
23	0	0	0	0	0	0
24	0	0	0	0	0	...

$ic := 0 .. 5$ $\quad f_{c_{ic}} := 10^{6} \cdot M_{1,ic}$ $\quad n_{cell_{ic}} := M_{2,ic}$ $\quad \beta_{opt_{ic}} := M_{3,ic}$ $\quad \beta_{g_{ic}} := M_{4,ic}$ $\quad \beta_{gs_{ic}} := \beta_{g_{ic}}$ $\quad \beta_{gs_4} := 0.64$ $\quad \beta_{gs_5} := 0.947$

$U_{max_{ic}} := 10^{6} \cdot M_{6,ic}$ $\quad G_{c_{ic}} := M_{9,ic}$ $\quad R_Q_{ic} := M_{10,ic}$ $\quad Q_{0_{ic}} := M_{11,ic}$ $\quad \delta f_{mf_{ic}} := M_{15,ic}$ $\quad \varphi_{acc_{ic}} := \frac{\pi}{180} \cdot M_{18,ic}$

$Leff := \overrightarrow{\left(\frac{1}{2 \cdot f_c} \cdot c \cdot \beta_{opt} \cdot n_{cell}\right)}$ $\quad Leff_4 := \frac{1}{2 \cdot f_{c_4}} \cdot c \cdot \beta_{g_4} \cdot n_{cell_4}$ $\quad Leff_5 := \frac{1}{2 \cdot f_{c_5}} \cdot c \cdot \beta_{g_5} \cdot n_{cell_5}$

$$E_{acc} := \overrightarrow{\left(\frac{U_{max}}{0.01 \cdot Leff}\right)} \qquad Epeak_Eacc_{ic} := \frac{M_{7,ic}}{E_{acc_{ic}} \cdot 10^{-6}} \qquad Bpeak_Eacc_{ic} := \frac{M_{8,ic}}{E_{acc_{ic}} \cdot 10^{-6}}$$

Optimal coupling for $I_{cpl} := 0.0026$ A:

$$\beta_{cpl_{ic}} := \sqrt{\left(1 + \frac{I_{cpl} \cdot \cos(\varphi_{acc_{ic}}) \cdot R_Q_{ic} \cdot Q_{0_{ic}}}{U_{max_{ic}}}\right)^2 + \left(\frac{2 \cdot \delta f_{mf_{ic}} \cdot Q_{0_{ic}}}{f_{c_{ic}}}\right)^2}$$

Loaded Q: $Q_L := \frac{Q_0}{1 + \beta_{cpl}}$

Required cavity power:

$$P_{req}(U_{max}, I_{linac}, \varphi_{acc}, R_Q, Q_0, \beta_{cpl}, f_c, \delta f) := \frac{(U_{max})^2 \cdot (1 + \beta_{cpl})^2}{4 \cdot \beta_{cpl} \cdot Q_0 \cdot R_Q} \cdot \left[\left[1 + \frac{I_{linac} \cdot \cos(\varphi_{acc}) \cdot R_Q \cdot Q_0}{U_{max} \cdot (1 + \beta_{cpl})}\right]^2 + \left(\frac{Q_0}{1 + \beta_{cpl}} \cdot \frac{2 \cdot \delta f}{f_c}\right)^2\right]$$

Energy stored in the cavity: $En_{cav} := \overrightarrow{\left(\frac{U_{max}^2}{R_Q} \cdot \frac{1}{2 \cdot \pi \cdot f_c}\right)}$

Surface resistance: $R_{s_{ic}} := \frac{G_{c_{ic}}}{Q_{0_{ic}}}$ $\qquad P_{r_{ic}} := P_{req}(U_{max_{ic}}, I_{linac}, \varphi_{acc_{ic}}, R_Q_{ic}, Q_{0_{ic}}, \beta_{cpl_{ic}}, f_{c_{ic}}, \delta f_{mf_{ic}})$

Generator power: $P_{g_{ic}} := \frac{P_{r_{ic}}}{M_{16,ic} \cdot M_{17,ic}}$

Cavity bandwidth: $\Delta f_{c_{ic}} := \frac{f_{c_{ic}}}{Q_{0_{ic}}} \cdot (1 + \beta_{cpl_{ic}})$

Cavity detuning: $\Delta f_{dtn_{ic}} := -f_{c_{ic}} \cdot \frac{I_{linac} \cdot \sin(\varphi_{acc_{ic}}) \cdot R_Q_{ic}}{2 \cdot U_{max_{ic}}}$

$$Leff = \begin{pmatrix} 12.361 \\ 20.663 \\ 20.478 \\ 43.816 \\ 70.336 \\ 106.08 \end{pmatrix} \text{cm} \qquad E_{acc} \cdot 10^{-6} = \begin{pmatrix} 0.809 \\ 9.679 \\ 10.011 \\ 11.382 \\ 16.919 \\ 18.759 \end{pmatrix} \text{MV/m} \qquad Epeak_Eacc = \begin{pmatrix} 7.416 \\ 4.643 \\ 3.836 \\ 3.514 \\ 2.4 \\ 2.074 \end{pmatrix} \qquad Bpeak_Eacc = \begin{pmatrix} 0 \\ 4.992 \\ 5.804 \\ 5.667 \\ 4.451 \\ 3.897 \end{pmatrix} \qquad En_{cav} = \begin{pmatrix} 0.019 \\ 14.403 \\ 8.504 \\ 41.007 \\ 101.683 \\ 158.958 \end{pmatrix} \text{J}$$

$$R_s \cdot 10^9 = \begin{pmatrix} 0 \\ 9.6 \\ 14 \\ 14.375 \\ 8.977 \\ 8.667 \end{pmatrix} \text{n}\Omega \qquad \beta_{cpl} = \begin{pmatrix} 1 \\ 2155.03 \\ 1985.033 \\ 1583.169 \\ 2078.372 \\ 3021.545 \end{pmatrix} \qquad P_g \cdot 10^{-3} = \begin{pmatrix} 2.622 \\ 6.576 \\ 6.138 \\ 17.002 \\ 39.29 \\ 64.392 \end{pmatrix} \text{kW} \qquad \overrightarrow{(I_{linac} \cdot U_{max} \cdot \cos(\varphi_{acc}))} \cdot 10^{-3} = \begin{pmatrix} 0 \\ 4 \\ 4.1 \\ 9.974 \\ 23.8 \\ 39.8 \end{pmatrix} \text{kW}$$

$$\Delta f_c = \begin{pmatrix} 30864.198 \\ 70.071 \\ 107.577 \\ 64.357 \\ 62.865 \\ 65.488 \end{pmatrix} \text{Hz} \qquad \Delta f_{dtn} = \begin{pmatrix} -817.375 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{Hz} \qquad Leff = \begin{pmatrix} 12.361 \\ 20.663 \\ 20.478 \\ 43.816 \\ 70.336 \\ 106.08 \end{pmatrix} \text{cm} \qquad \frac{Q_L}{10^6} = \begin{pmatrix} 5.265 \times 10^{-3} \\ 2.319 \\ 3.021 \\ 5.05 \\ 10.34 \\ 9.925 \end{pmatrix} \qquad \begin{pmatrix} \text{MEBT} \\ \text{HWR} \\ \text{SSR1} \\ \text{SSR2} \\ \text{LB650} \\ \text{HB650} \end{pmatrix}$$

$$\Delta f_{LFD_{ic}} := M_{21,ic} \cdot \left(10^{-4} \cdot \frac{U_{max_{ic}}}{Leff_{ic}}\right)^2$$

$$\Delta f_{LFD} = \begin{pmatrix} 0 \\ 0 \\ 501.068 \\ 362.723 \\ 406.469 \\ 351.913 \end{pmatrix} \text{Hz} \qquad \overrightarrow{\left(\frac{\Delta f_{LFD}}{\Delta f_c}\right)} = \begin{pmatrix} 0 \\ 0 \\ 4.658 \\ 5.636 \\ 6.466 \\ 5.374 \end{pmatrix}$$

Required cavity power for 5 mA operation ($I_L := 0.005$ A):

$$P_{r5_{ic}} := P_{req}\left(U_{max_{ic}}, I_L, \varphi_{acc_{ic}}, R_Q_{ic}, Q_{0_{ic}}, \beta_{cpl_{ic}}, f_{c_{ic}}, \delta f_{mf_{ic}}\right)$$

$$\frac{P_{r5}}{10^3} = \begin{pmatrix} 1.888 \\ 11.049 \\ 11.336 \\ 27.606 \\ 65.924 \\ 110.079 \end{pmatrix} \text{kW}$$

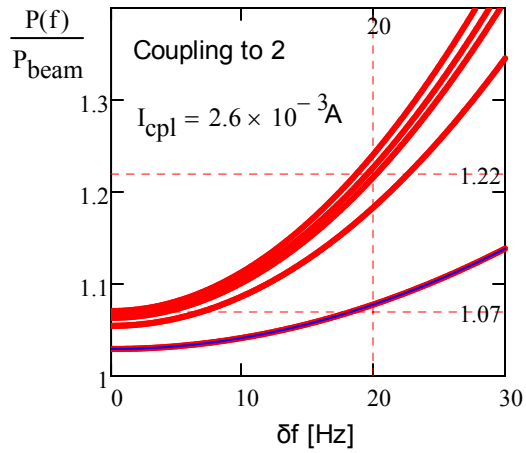
Ratio of cavity power to the beam power: $\overrightarrow{\frac{P_{r5}}{(I_L \cdot U_{max})}} = \begin{pmatrix} 3.776 \\ 1.105 \\ 1.106 \\ 1.107 \\ 1.108 \\ 1.106 \end{pmatrix}$

$\delta f := 0, 0.1 .. 30$ $\qquad$ $icc := 2$

BCS resiistance estimate:

$f := 0.65 \cdot 10^9$ $\qquad$ $T_c := 2$ K

$$R_{BCS} := \frac{2 \cdot 10^{-4}}{T_c} \cdot \left(\frac{f}{1.5 \cdot 10^9}\right)^2 \cdot \exp\left(\frac{-17.67}{T_c}\right) \qquad R_{BCS} = 2.733 \times 10^{-9}\,\Omega$$



Flight factors

$$\kappa_f(\beta, \beta_w, n_{cell}) := \frac{2 \cdot \beta}{\pi \cdot n_{cell}} \cdot \left[\frac{\sin\left(\frac{\beta - \beta_w}{\beta} \cdot \frac{\pi \cdot n_{cell}}{2}\right)}{\beta - \beta_w} + (-1)^{n_{cell}+1} \cdot \frac{\sin\left(\frac{\beta + \beta_w}{\beta} \cdot \frac{\pi \cdot n_{cell}}{2}\right)}{\beta + \beta_w}\right] \cdot \left[0.9599 \cdot (n_{cell} < 3) + 0.9939 \cdot (n_{cell} > 3)\right]$$

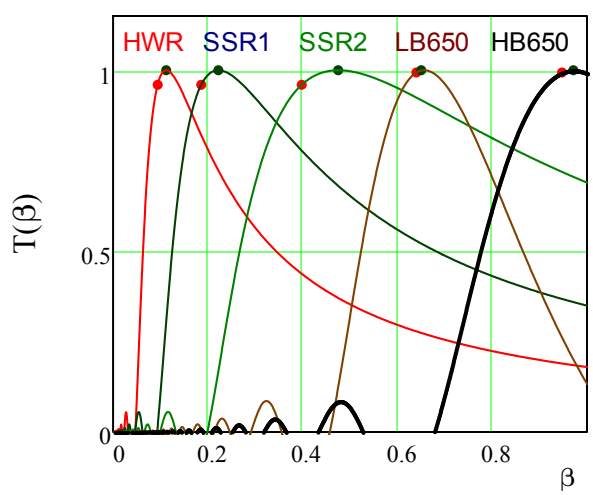
$ic1 := 1 .. 5$ $\qquad$ $\kappa f_{ic1} := \kappa_f\left(\beta_{gs_{ic1}} + 10^{-8}, \beta_{gs_{ic1}}, n_{cell_{ic1}}\right)$

Cavity filling time for operation in pulsed regime

Decay time for amplitude: $\qquad \tau_d := \overrightarrow{\frac{2 \cdot Q_L}{2 \cdot \pi \cdot f_c}}$

Filling time: $\tau_{fill} := \ln(2) \cdot \tau_d$

$$\tau_d \cdot 10^3 = \begin{pmatrix} 0.0103 \\ 4.5427 \\ 2.9589 \\ 4.946 \\ 5.0634 \\ 4.8605 \end{pmatrix} \text{ms} \qquad \tau_{fill} \cdot 10^3 = \begin{pmatrix} 0.0071 \\ 3.1487 \\ 2.051 \\ 3.4283 \\ 3.5097 \\ 3.3691 \end{pmatrix} \text{ms}$$



Real part of optimal beam current in the abcence of detuning

$$Re_I_{opt_{ic}} := \frac{\beta_{cpl_{ic}} - 1}{R_Q_{ic} \cdot Q_{0_{ic}}} \cdot U_{max_{ic}}$$

$$Re_I_{opt} \cdot 10^3 = \begin{pmatrix} 0 \\ 3.168 \\ 2.801 \\ 3.321 \\ 3.372 \\ 3.285 \end{pmatrix} \text{ mA}$$

$$\overrightarrow{\left(\frac{P_r}{I_{linac} \cdot U_{max}}\right)} = \begin{pmatrix} 9.44 \\ 1.184 \\ 1.078 \\ 1.227 \\ 1.241 \\ 1.217 \end{pmatrix}$$

$$\frac{Re_I_{opt}}{I_{linac}} = \begin{pmatrix} 0 \\ 1.584 \\ 1.401 \\ 1.66 \\ 1.686 \\ 1.642 \end{pmatrix}$$

Cryogenic power

$$P_{cav} := \frac{\left(\overrightarrow{U_{max}}\right)^2}{R_Q \cdot Q_0} \qquad P_{crm_{ic}} := P_{cav_{ic}} \cdot M_{12,ic} \qquad P_{TotPerCrm_{ic}} := M_{14,ic} + P_{crm_{ic}} \qquad P_{cav} = \begin{pmatrix} 1888.007 \\ 2.941 \\ 2.894 \\ 10.467 \\ 19.315 \\ 21.64 \end{pmatrix} W \qquad P_{crm} = \begin{pmatrix} 1888.007 \\ 23.529 \\ 23.154 \\ 52.336 \\ 57.946 \\ 129.839 \end{pmatrix} W$$

$$P_{StatTot} := \sum_{ic=1}^{5} \left(M_{14,ic} \cdot M_{13,ic}\right) \qquad P_{DynTotCW} := \sum_{ic=1}^{5} \left(P_{crm_{ic}} \cdot M_{13,ic}\right)$$

$$P_{CryoTotCW} := P_{StatTot} + P_{DynTotCW} \qquad P_{StatTot} = 164 \ W \qquad P_{DynTotCW} = 1592.953 W \qquad P_{CryoTotCW} = 1756.95 W$$

Total static loads to 5 and 70 K

$$P_{StatTot5K} := \sum_{ic=1}^{5} \left(M_{19,ic} \cdot M_{13,ic}\right) = 958 \ W \qquad P_{StatTot70K} := \sum_{ic=1}^{5} \left(M_{20,ic} \cdot M_{13,ic}\right) = 2514 \ W \qquad P_{TotPerCrm} = \begin{pmatrix} 1.888 \times 10^3 \\ 60.529 \\ 36.154 \\ 61.336 \\ 59.946 \\ 133.839 \end{pmatrix} W$$

Pulsed operation

$$\int_0^{\ln(2)\cdot\tau} \left[2 \cdot \left(1 - \exp\left(-\frac{t}{\tau_d}\right)\right)\right]^2 dt = \tau \cdot \left(\ln(16) - \frac{5}{2}\right) \quad \Rightarrow \quad F_{cryo} := \frac{\ln(16) - 2.5}{\ln(2)} = 0.393$$

Accelerated cavity discharge: $E_{discharge}(t,\tau) := \exp\left(-\frac{t}{\tau}\right) - 2 \cdot \left(1 - \exp\left(-\frac{t}{\tau}\right)\right) \qquad F_{dschg} := \frac{1}{0.40547} \cdot \int_0^{0.40547} E_{discharge}(t,1)^2 \, dt = 0.301$

$$\eta_{cryo} := \left(\tau_{fill} \cdot F_{cryo} + F_{dschg} \cdot \tau_d \cdot 0.40547 + T_{inj}\right) \cdot f_{rep}$$

$$P_{CryoTotDynamic} := P_{crm_1} \cdot M_{13,1} + \sum_{ic=2}^{5} \left(\eta_{cryo_{ic}} \cdot P_{crm_{ic}} \cdot M_{13,ic}\right) = 101.133 \ W$$

$$P_{CryoTotPulsed} := P_{StatTot} + P_{CryoTotDynamic} = 265.133 \ W$$

$$\sum_{ic=2}^{5} \left(\eta_{cryo_{ic}} \cdot P_{crm_{ic}} \cdot M_{13,ic}\right) = 77.604 \ W$$

$$\eta_{cryo} = \begin{pmatrix} 1.082 \\ 4.658 \\ 3.408 \\ 4.976 \\ 5.068 \\ 4.908 \end{pmatrix} \cdot \%$$

$\eta_{cryo_{ic}} \cdot P_{crm_{ic}} =$

20.427
1.096
0.789
2.604
2.937
6.373

$\frac{W}{crm}$

Normal cavity discharge: $F_{dschgN} = \frac{1}{\tau_d} \cdot \int_0^{\infty} \exp\left(-\frac{t}{\tau_d}\right)^2 dt \qquad F_{dschgN} := \frac{1}{2}$

$$\eta_{cryoN} := \left(\tau_{fill} \cdot F_{cryo} + F_{dschgN} \cdot \tau_d + T_{inj}\right) \cdot f_{rep}$$

$$\eta_{cryoN} = \begin{pmatrix} 0.011 \\ 0.081 \\ 0.056 \\ 0.087 \\ 0.089 \\ 0.086 \end{pmatrix}$$

$$P_{CryoTotDynamicN} := P_{crm_1} \cdot M_{13,1} + \sum_{ic=2}^{5} \left(\eta_{cryoN_{ic}} \cdot P_{crm_{ic}} \cdot M_{13,ic}\right) = 159.373 \ W$$

$$P_{CryoTotPulsedN} := P_{StatTot} + P_{CryoTotDynamicN} = 323.373 \ W$$

$$\sum_{ic=1}^{5} \left(\eta_{cryoN_{ic}} \cdot P_{crm_{ic}} \cdot M_{13,ic}\right) = 137.748 \ W$$

$$\left(\tau_{fill} + T_{inj}\right) \cdot f_{rep} = \begin{pmatrix} 0.011 \\ 0.074 \\ 0.052 \\ 0.079 \\ 0.081 \\ 0.078 \end{pmatrix}$$

$\eta_{cryoN_{ic}} \cdot P_{crm_{ic}} =$

20.574
1.904
1.307
4.562
5.156
11.146

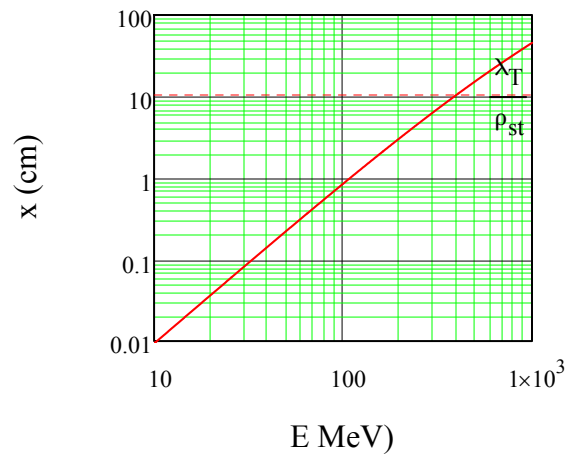
$\frac{W}{crm}$

$$\frac{\sum_{ic=1}^{5} \left(\eta_{cryo_{ic}} \cdot P_{crm_{ic}} \cdot M_{13,ic}\right)}{\sum_{ic=1}^{5} \left(\eta_{cryoN_{ic}} \cdot P_{crm_{ic}} \cdot M_{13,ic}\right)} = 0.571 \qquad \frac{P_{CryoTotPulsedN}}{P_{CryoTotPulsed}} = 1.22 \qquad \frac{P_{CryoTotPulsedN}}{P_{CryoTotCW}} = 0.184$$

Required thickness of linac collimators

Collimator material steel => $\rho_{st} := 7.8$ g/cm^3 $\quad dE_dx_0 := 1.45\cdot 10^6$ eV/(g/cm^2) $\quad \lambda_T := 81.7$ g/cm^2

Stopping length: $$x(E) := \frac{E^2}{(E + m_p)\cdot dE_dx_0\cdot \rho_{st}}$$



Simple estimate of beam acceleration

$E_0 := 2.1\cdot 10^6$ eV

$EnD(E_0) :=$

$i \leftarrow 0$

$Ncrm \leftarrow 0$

$E_0 \leftarrow E_0$

for $ic \in 1..5$

  for $ncrm \in 0..M_{13,ic} - 1$

    for $ncav \in 0..M_{12,ic} - 1$

$$\beta_i \leftarrow \sqrt{1 - \frac{m_p^2}{(E_i + m_p)^2}}$$

$$\varphi_i \leftarrow 6 + \frac{65}{(i+1)^{0.344}}$$

$$\kappa f_i \leftarrow \kappa_f(\beta_i, \beta_{g_{ic}}, n_{cell_{ic}})$$

$$E_{i+1} \leftarrow E_i + U_{max_{ic}}\cdot \kappa f_i\cdot \cos\left(\frac{\pi}{180}\cdot \varphi_i\right)$$

$$\Delta E_i \leftarrow U_{max_{ic}}\cdot \kappa f_i$$

$i \leftarrow i + 1$

$Ncrm_i \leftarrow ncrm + 1$

$$\beta_i \leftarrow \sqrt{1 - \frac{m_p^2}{(E_i + m_p)^2}}$$

$\varphi_i \leftarrow \varphi_{i-1}$

$Ncrm_i \leftarrow Ncrm_{i-1}$

$\kappa f_i \leftarrow \kappa f_{i-1}$

$\Delta E_i \leftarrow \Delta E_{i-1}$

$augment(E, Ncrm, \kappa f, \beta, \varphi, \Delta E)$

$$N_{cav} := \sum_{ic=1}^{5} (M_{12,ic}\cdot M_{13,ic}) = 116 \qquad ncav := 0..N_{cav}$$

$En := EnD(E_0)$

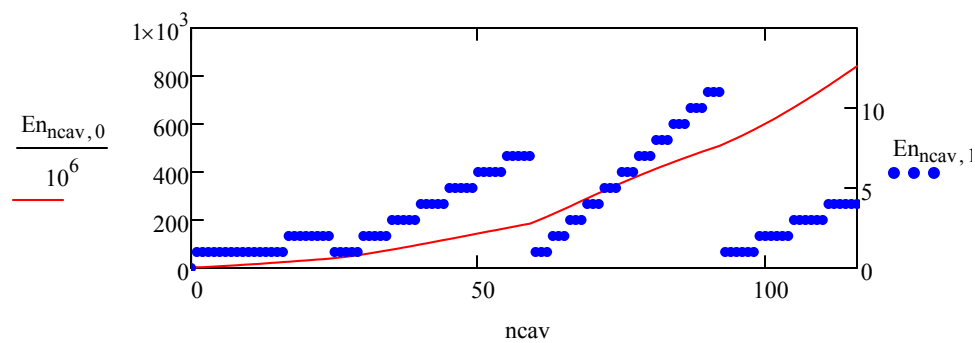
$n_{end_0} := 0 \qquad n_{end_{ic+1}} := n_{end_{ic}} + M_{12,ic+1}\cdot M_{13,ic+1}$

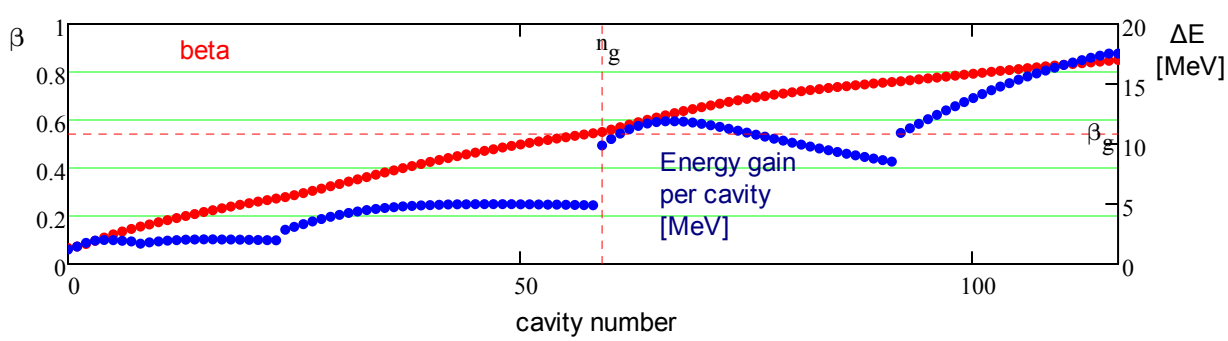
$n_g := 59$

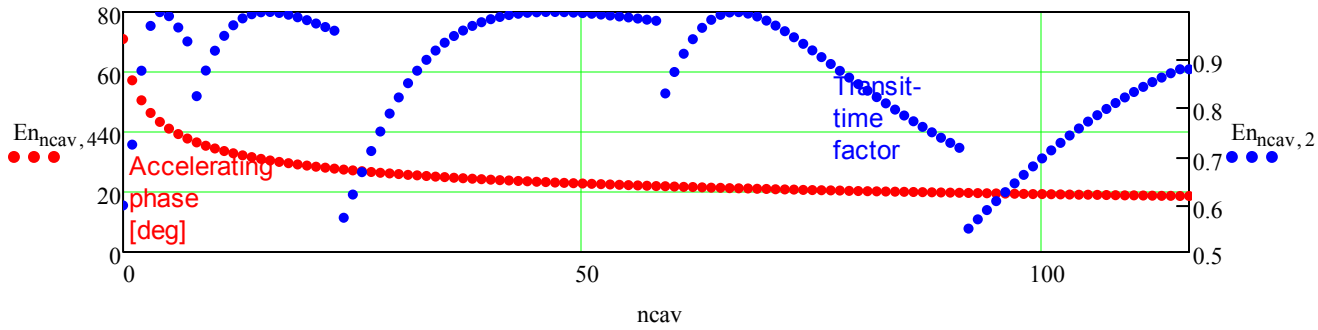
$\beta_g := En_{n_g,3}$

$n_{end_{ic}} =$	$En_{n_{end_{ic}},0}\cdot 10^{-6} =$	$En_{n_{end_{ic}},3} =$
0	2.1	0.067
8	11.658	0.156
24	38.633	0.278
59	184.995	0.55
92	508.885	0.761
116	841.437	0.85

MeV






80
60
40
20
0
0
50
100
ncav
0.9
0.8
0.7
0.6
0.5
$En_{ncav,4}$
$En_{ncav,2}$
Accelerating
phase
[deg]
Transit-
time
factor

Booster

Circumference: $C := 47420.214$ cm

Injection energy: $E_{min} := 0.8 \cdot 10^9$ eV $\qquad P_{min} := \sqrt{(E_{min} + m_p)^2 - m_p^2}$

Injection momentum: $P_{min} = 1.463 \times 10^9$

Extraction energy: $E_{max} := 8 \cdot 10^9$ eV $\qquad P_{max} := \sqrt{(E_{max} + m_p)^2 - m_p^2}$

Extraction momentum: $P_{max} = 8.889 \times 10^9$ eV

Ramp frequency: $f_{ramp} := f_{rep} = 20$ Hz

Harmonic number $q_b := 84$

Momentum compaction: $\alpha := .03346$ $\qquad \gamma_t := \frac{1}{\sqrt{\alpha}}$

Transition gamma: $\gamma_t = 5.467$

Hor norm. emittance: $\varepsilon_{nx95} := 0.0016$ cm

Vert norm. emittance: $\varepsilon_{ny95} := 0.0016$ cm

Hor effective norm. emit.: $\varepsilon_{nxEf} := 0.0005$ cm

Hor effective norm. emit.: $\varepsilon_{nyEf} := 0.0005$ cm

\ These two emittances are determined so that the beam density in the
| beam center obtained in the strip- injection simulations coinside with the
/ beam density of a Gaussian beam with these emittances

Minimum L. bucket size (total aria): $\varepsilon_{LB} := 0.077$ eV*s

RMS L. emittance(Δx Δp) at injection: $\varepsilon_{LB\sigma} := 3.2 \cdot 10^{-3}$ eV*s

Beam acceleration

$N_{turn} \equiv 15151 \qquad n_{tr} \equiv 7136 \qquad n := 0 .. N_{turn}$

$$n_b \equiv (0000 \;\; 260.0 \;\; 1200 \;\; 4000 \;\; 6000 \;\; 6800 \;\; 7000 \;\; 7150 \;\; 7400 \;\; 7900 \;\; 10500 \;\; 12000 \;\; 14000 \;\; N_{turn})^T$$

$$V_b \equiv 10^6 \cdot (0.204 \;\; 0.36 \;\; 0.700 \;\; 0.980 \;\; 1.05 \;\; 1.07 \;\; 1.08 \;\; 1.12 \;\; 1.17 \;\; 1.180 \;\; 1.170 \;\; 1.12 \;\; 0.900 \;\; 0.45)^T$$

$vs := cspline(n_b, V_b) \qquad V0_n := interp(vs, n_b, V_b, n)$

$$P(t) := P_{min} + \frac{P_{max} - P_{min}}{2} \cdot (1 - \cos(2 \cdot \pi \cdot f_{ramp} \cdot t)) \qquad En(t) := \sqrt{P(t)^2 + m_p^2} - m_p \qquad v(t) := c \cdot \frac{P(t)}{m_p + En(t)}$$

$$t_0 := 0 \qquad t_{n+1} := t_n + \frac{C}{v(t_n)} \qquad \beta_n := \frac{v(t_n)}{c} \qquad p_n := P(t_n) \qquad f0_n := \frac{v(t_n)}{C}$$

$t_{N_{turn}} \cdot 10^3 = 25.001$ ms

$$\gamma_n := 1 + \frac{En(t_n)}{m_p} \qquad f_{RF_n} := q_b \cdot f0_n \qquad E_n := En(t_n)$$

$\frac{10^3}{2 \cdot f_{ramp}} = 25$ ms

$$\eta s_n := \alpha - \frac{1}{(\gamma_n)^2} \qquad \nu_{s0_n} := \sqrt{\frac{V0_n \cdot q_b \cdot |\eta s_n|}{2 \cdot \pi \cdot m_p \cdot \gamma_n \cdot (\beta_n)^2}}$$

$n := 0 .. N_{turn} - 1 \qquad Vacc_n := E_{n+1} - E_n \qquad \phi 0_n := asin\left(\frac{Vacc_n}{V0_n}\right) \qquad \nu_{s_n} := \nu_{s0_n} \cdot \sqrt{\cos(\phi 0_n)}$

$$\varepsilon_{Laccep_n} := \frac{8}{\pi} \cdot \frac{m_p \cdot C}{c \cdot q_b} \cdot \sqrt{\frac{V0_n \cdot \gamma_n}{2 \cdot \pi \cdot m_p \cdot q_b \cdot |\eta s_n|}} \cdot \frac{1 - \sin(\phi 0_n)}{(1 + 0.5 \cdot \sin(\phi 0_n))^2} \qquad \Delta p_p_{max_n} := \frac{2 \cdot \nu_{s0_n}}{q_b \cdot |\eta s_n|} \cdot \sqrt{\cos(\phi 0_n) - \left(\frac{\pi}{2} - \phi 0_n\right) \cdot \sin(\phi 0_n)}$$

$\phi 0a_n := \phi 0_n \cdot (n < n_{tr}) + (\pi - \phi 0_n) \cdot (n \geq n_{tr})$

$\sigma\varphi_n := q_b \cdot \sqrt{\dfrac{2 \cdot \pi \cdot \varepsilon_{LB\sigma} \cdot c \cdot |\eta s_n|}{m_p \cdot C \cdot \beta_n \cdot \gamma_n \cdot \nu_{s_n}}}$

$\sigma\varphi_n := q_b \cdot \sqrt{\dfrac{2 \cdot \pi \cdot \varepsilon_{LB\sigma} \cdot c \cdot |\eta s_n|}{m_p \cdot C \cdot \beta_n \cdot \gamma_n \cdot \nu_{s_n}}}$

$\sigma p_n := \sqrt{\dfrac{2 \cdot \pi \cdot \varepsilon_{LB\sigma} \cdot c \cdot \nu_{s_n}}{m_p \cdot C \cdot \beta_n \cdot \gamma_n \cdot |\eta s_n|}}$

$\sigma p_n := \sqrt{\dfrac{2 \cdot \pi \cdot \varepsilon_{LB\sigma} \cdot c \cdot \nu_{s_n}}{m_p \cdot C \cdot \beta_n \cdot \gamma_n \cdot |\eta s_n|}}$

$B_n := \dfrac{\sqrt{2 \cdot \pi}}{\sigma\varphi_n}$

L. emit. jump at transition: $\kappa_{Ltr} := 72\%$

RMS L. emittance(Δx Δp) at extraction: $\varepsilon_{LB\sigma} := 3.2 \cdot 10^{-3}$ eV*s

MI 97% L. acceptance for slip-stacking: $\varepsilon_{Lmax} := 0.1$ eV s

- It implies the Recycler bucket of 4.2 ns, 8 MeV:

$(\pi \cdot 8 \cdot 4.2 \cdot 10^{-3} = 0.106)$ eV s

Maxumum RF voltage: $\max(V0) \cdot 10^{-6} = 1.184$ MV

RF frequency at injection: $f_{RF_0} \cdot 10^{-6} = 44.705$ MHz

RF frequency at extraction: $f_{RF_{N_{turn}}} \cdot 10^{-6} = 52.812$ MHz

$\varepsilon x_n := \dfrac{\varepsilon_{nxEf}}{\beta_n \cdot \gamma_n}$ $\varepsilon y_n := \dfrac{\varepsilon_{nyEf}}{\beta_n \cdot \gamma_n}$

Number of particles at extraction: $N_{be} := 6.437 \cdot 10^{12}$

Booster efficiency in acceleration: $\kappa_b := 0.98$

Efficiency of Booster injection: $\kappa_{bi} := 0.98$

$N_{bi} := \dfrac{N_{be}}{\kappa_b}$

Number of particles at injection: $N_{bi} = 6.568 \times 10^{12}$

$N_{turnB} := \dfrac{e_{SI} \cdot N_{bi} \cdot f0_0}{\kappa_{bi} \cdot I_{linac}}$ $T_{inj} := \dfrac{N_{turnB}}{f0_0}$

Number of injection turns: $N_{turnB} = 285.735$

Injection time $T_{inj} \cdot 10^3 = 0.537$ ms

Number of bunches: $n_b := 81$

$N_{bb} := \dfrac{N_{bi}}{n_b}$ $Ib_n := e_{SI} \cdot N_{bi} \cdot f0_n$

Bunch population at injection: $N_{bb} = 8.109 \times 10^{10}$

$P_B := e_{SI} \cdot N_{be} \cdot E_{max} \cdot f_{ramp}$

Beam power: $P_B \cdot 10^{-3} = 165.003$ kW

Injection beam current: $Ib_0 = 0.56$ A

Extraction beam current: $Ib_{N_{turn}-1} \cdot \kappa_b = 0.648$ A

Booster acceptance: $\varepsilon_{accept} := 35 \cdot 10^{-4}$ cm

Normalized acceptance: $\varepsilon_{accept} \cdot \beta_0 \cdot \gamma_0 \cdot 10^4 = 54.585$

Booster aperture expressed in 95% beam size at injection: $\sqrt{\dfrac{\beta_0 \cdot \gamma_0 \cdot \varepsilon_{accept}}{\varepsilon_{ny95}}} = 1.847$

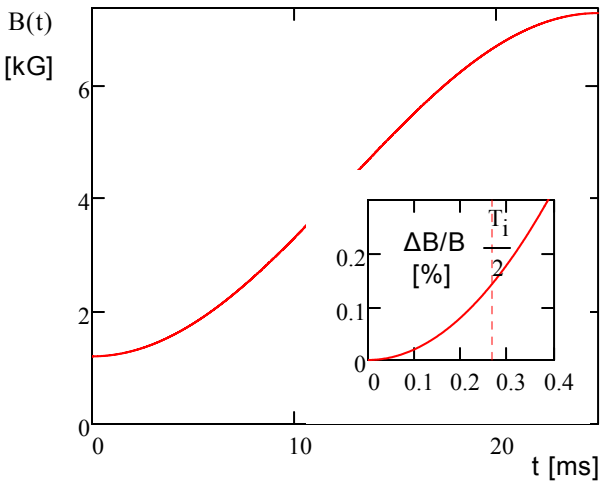
$\Delta p_p_{inj} := \dfrac{P_{max} - P_{min}}{2 \cdot P_{min}} \cdot (1 - \cos(\pi \cdot f_{ramp} \cdot T_{inj})) = 1.444 \times 10^{-3}$

Momentum change during injection: $\Delta p_p_{inj} = 1.444 \times 10^{-3}$

$\Sigma\nu_{s_0} := 0$ $\Sigma\nu_{s_{n+1}} := \Sigma\nu_{s_n} + \nu_{s_n}$

Corresponding change in the Booster correctors:

Number of correctors: $N_{cor} := 48$ Transfer function for horizontal correctors: $\alpha_{corH} := 366$ G-cm/A

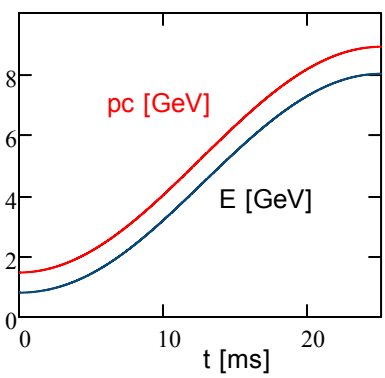


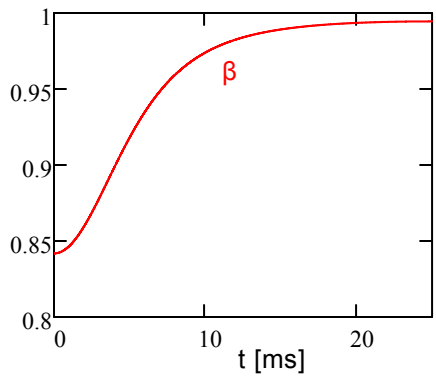
$T_i := T_{inj} \cdot 10^3$

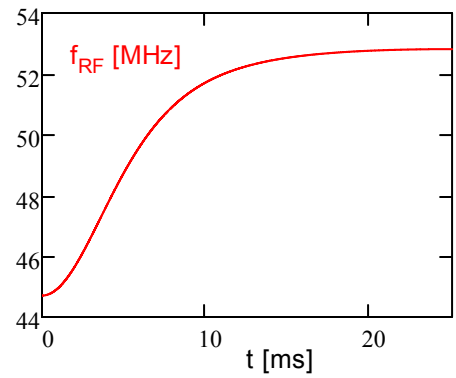
$$I_{cor} := \frac{2 \cdot \pi \cdot P_{min}}{e_{conv}} \cdot \frac{\Delta p_p_{inj}}{N_{cor} \cdot \alpha_{corH}} \qquad I_{cor} = 2.52 \quad A$$

Note that the maximum corrector current is 40 A.

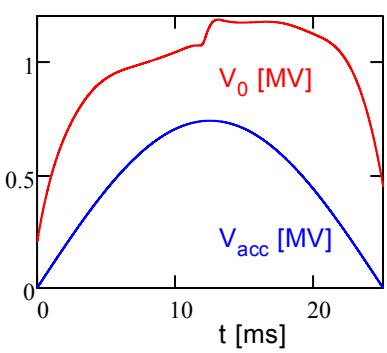
<= 7.200 GeV: 6.60 kG for F dipoleand 5.61 kG for D dipole



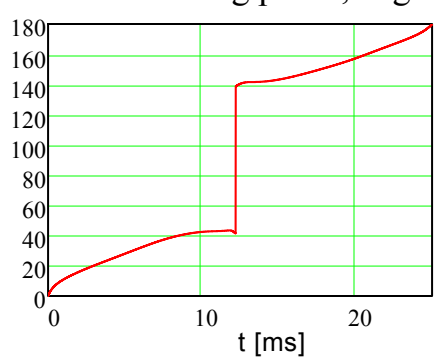




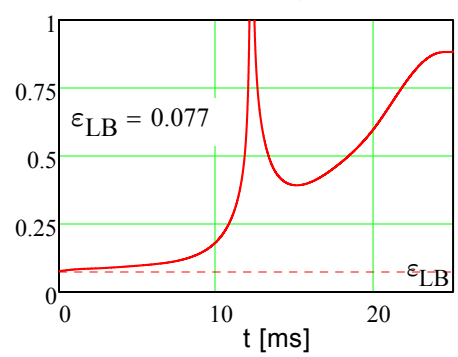
RF voltage



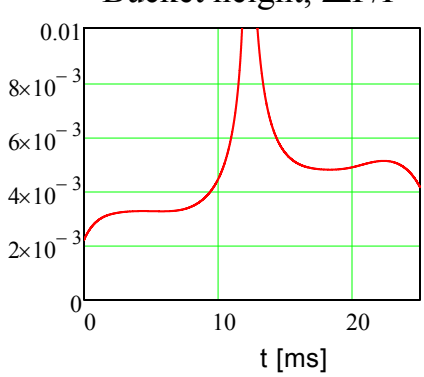
Accelerating phase, deg



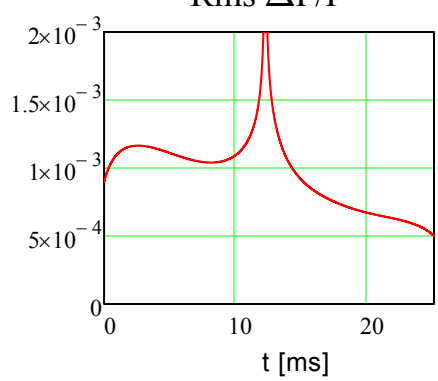
Bucket area, eV s



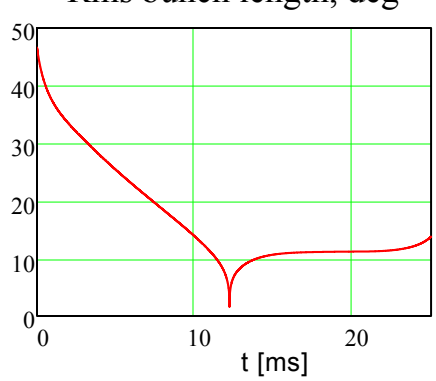
Bucket height, ΔP/P

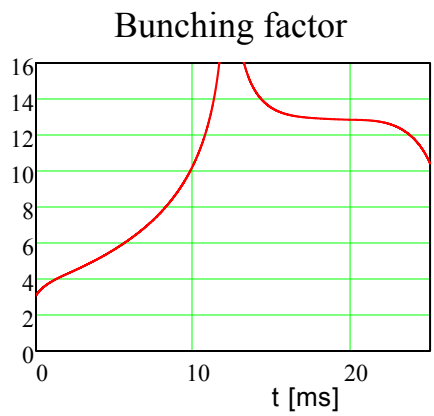


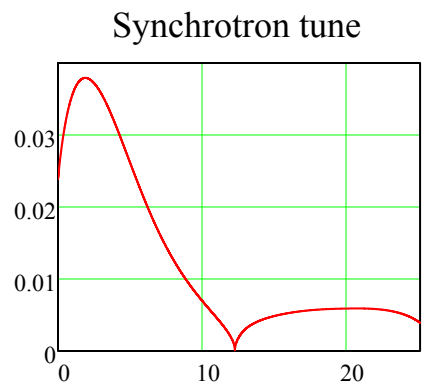
Rms ΔP/P

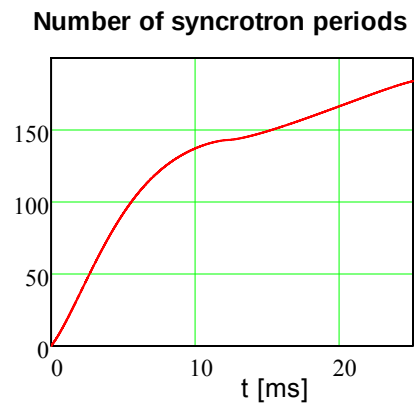


Rms bunch length, deg







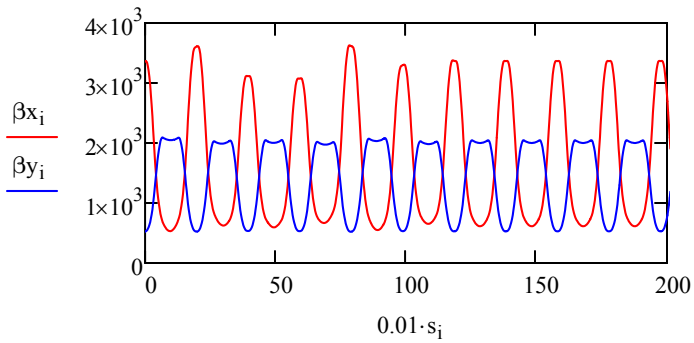


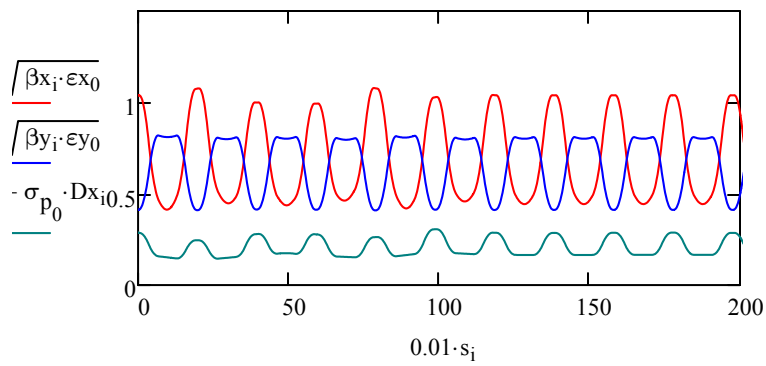
Ring Twiss parameters

$M :=$ BoosterTwiss.dat $\qquad N := \mathrm{rows}(M) \qquad i := 0 .. N-1 \qquad N = 2.946 \times 10^{3}$

$s_i := M_{i,2} \qquad \beta x_i := M_{i,3} \qquad \beta y_i := M_{i,5} \qquad Dx_i := M_{i,7} \qquad \nu x_i := M_{i,11} \qquad \nu y_i := M_{i,12}$

$\alpha x_i := M_{i,4} \qquad \alpha y_i := M_{i,6} \qquad Dxp_i := M_{i,8} \qquad \nu_x := \nu x_{N-1} \qquad \nu_y := \nu y_{N-1} \qquad \nu_x = 6.725 \qquad \nu_y = 6.798$





$nn := 0, 50 .. N_{turn} - 1$

$$\delta\nu_{ySC_{nn}} := \frac{r_p \cdot N_{bi} \cdot B_{nn}}{2 \cdot \pi \cdot (\beta_{nn})^2 \cdot (\gamma_{nn})^3 \cdot C} \cdot \sum_{j=0}^{N-2} \frac{\beta y_j \cdot (s_{j+1} - s_j)}{\sqrt{\beta y_j \cdot \varepsilon y_{nn}} \cdot \left[\sqrt{\beta x_j \cdot \varepsilon x_{nn} + (\sigma p_{nn} \cdot Dx_j)^2} + \sqrt{\beta y_j \cdot \varepsilon y_{nn}}\right]}$$

$$\delta\nu_{xSC_{nn}} := \frac{r_p \cdot N_{bi} \cdot B_{nn}}{2 \cdot \pi \cdot (\beta_{nn})^2 \cdot (\gamma_{nn})^3 \cdot C} \cdot \sum_{j=0}^{N-2} \frac{\beta x_j \cdot (s_{j+1} - s_j)}{\sqrt{\beta x_j \cdot \varepsilon x_{nn} + (\sigma p_{nn} \cdot Dx_j)^2} \cdot \left[\sqrt{\beta x_j \cdot \varepsilon x_{nn} + (\sigma p_{nn} \cdot Dx_j)^2} + \sqrt{\beta y_j \cdot \varepsilon y_{nn}}\right]}$$

Maximum H. SC tune shift for Gaussian tr. distribution: $\max(\delta\nu_{xSC}) = 0.191$

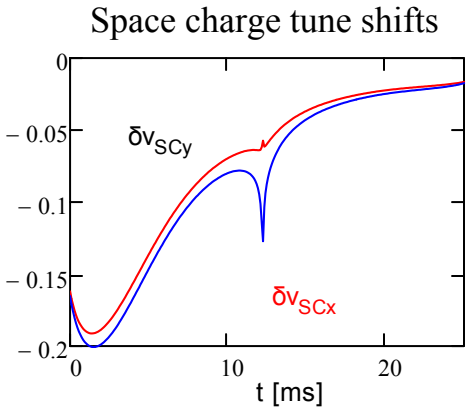
Maximum V. SC tune shift for Gaussian tr. distribution: $\max(\delta\nu_{ySC}) = 0.201$

Maximum H. SC tune shift for KV tr. distribution: $3^{-1} \cdot \max(\delta\nu_{xSC}) = 0.064$

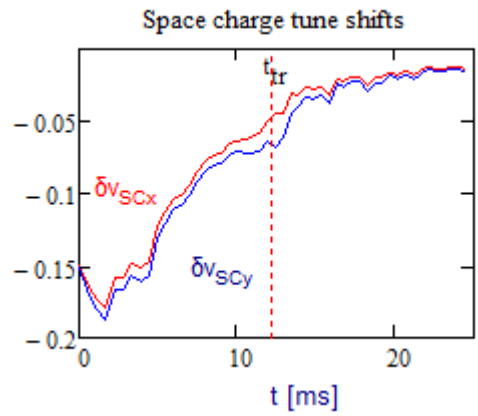
Maximum V. SC tune shift for KV tr. distribution: $3^{-1} \cdot \max(\delta\nu_{ySC}) = 0.067$

- It implies the same 100% emittance as 95% emittance for the Gaussian transverse distribution

The below picture was computed using multiparticle tracking for beam



acceleration



Cavity parameters

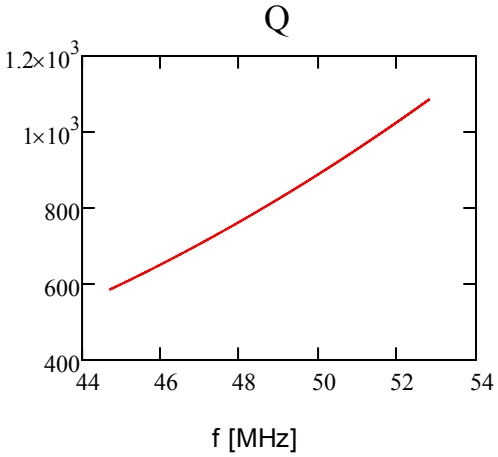
Number of cavities: $N_{cavB} := 22$

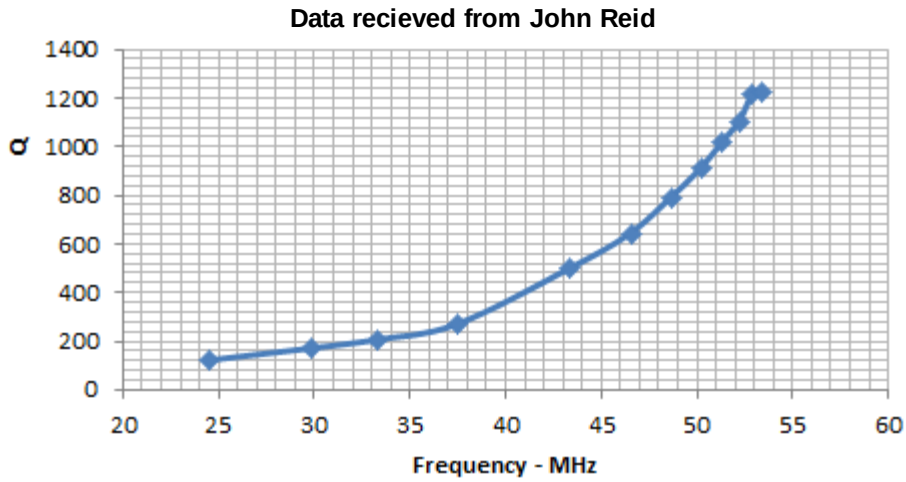
External Q-value: $Q_{ext} := 600$

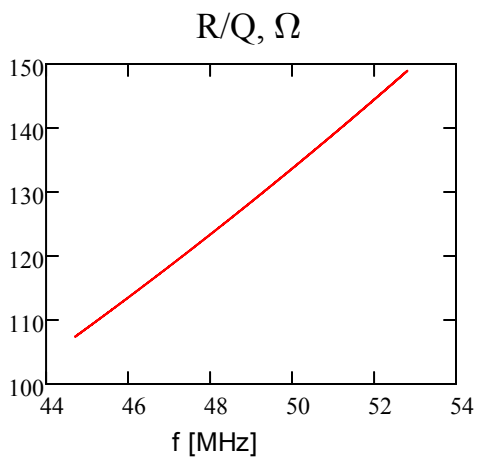
$$R_Q(f) := 75 + 75\cdot\left[0.8\cdot\frac{f - 37\cdot10^6}{(53 - 37)\cdot10^6} + 0.2\cdot\left[\frac{f - 37\cdot10^6}{(53 - 37)\cdot10^6}\right]^2\right]$$

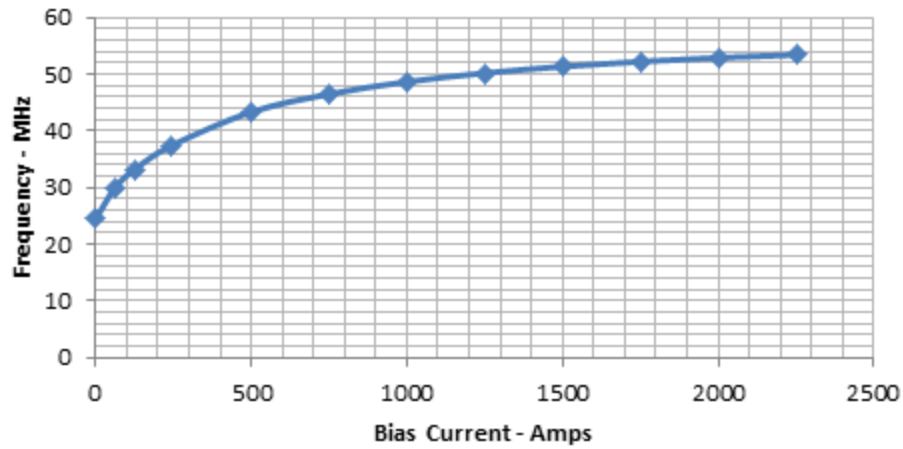
$$Q(f) := 300 + 800\cdot\left[\frac{f - 37\cdot10^6}{(53 - 37)\cdot10^6}\cdot0.5 + \left[\frac{f - 37\cdot10^6}{(53 - 37)\cdot10^6}\right]^2\cdot0.5\right]$$

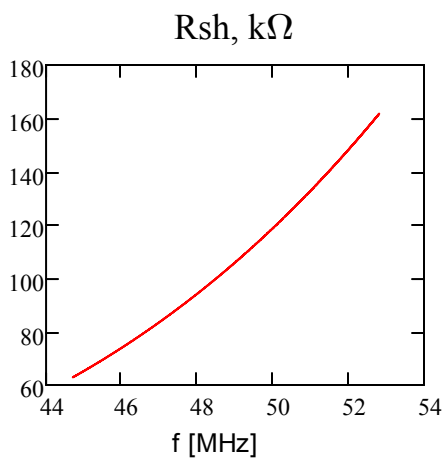
$$R_{sh}(f) := Q(f)\cdot R_Q(f) \qquad R_{sht}(f) := \frac{Q(f)\cdot Q_{ext}}{Q(f) + Q_{ext}}\cdot R_Q(f) \qquad P_{cavB_n} := \frac{1}{R_{sh}\left(f_{RF_n}\right)}\cdot\left(\frac{V0_n}{N_{cavB}}\right)^2 \qquad P_{acc_n} := \frac{Vacc_n}{N_{cavB}}\cdot Ib_n$$

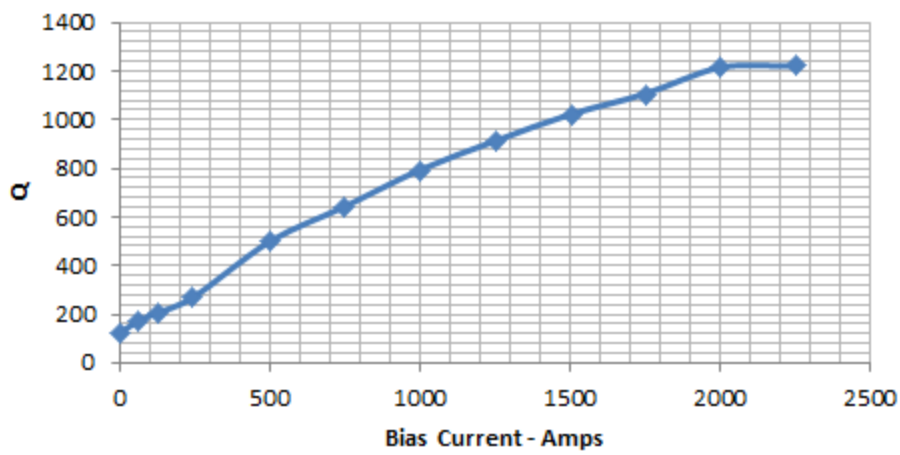




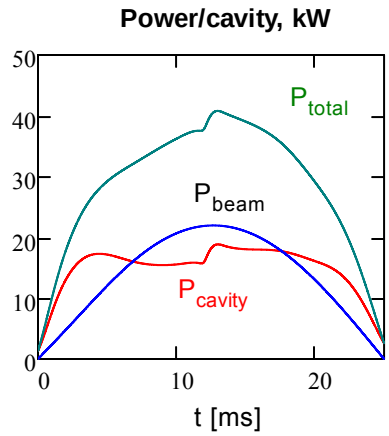


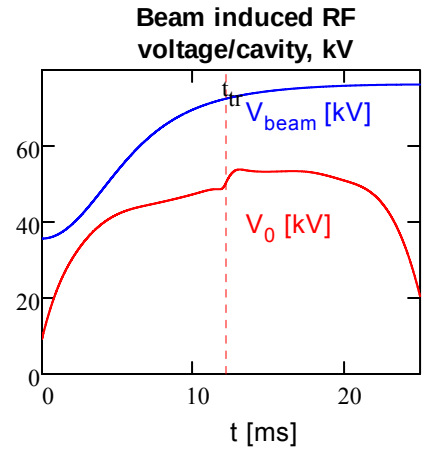






$t_{tr} := 10^3 \cdot t_{n_{tr}}$





Second order slip factor

$\beta_{xSF} := 3370$ cm $\qquad \beta_{xSD} := 716$ cm

$\beta_{ySF} := 525$ cm $\qquad \beta_{ySD} := 2048$ cm

$D_{SF} := 314$ cm $\qquad D_{SD} := 180.8$ cm

The below data were computed by A. Valishev with MADX simulations in August 2017

$$MS := \begin{array}{c} \begin{array}{ccc} Cx & alpha_c_p & Cy \end{array} \\ \begin{pmatrix} 6.30 & 0.0034 & 6.61 \\ 9.63 & -0.0272 & 7.37 \\ 12.97 & -0.0580 & 8.13 \\ 16.3 & -0.0887 & 8.90 \\ 19.6 & -0.1195 & 9.66 \\ 22.99 & -0.1502 & 10.42 \end{pmatrix} \end{array}$$

$k := 0 .. rows(MS) - 1$

Introduce $\quad A_F = \frac{e}{p \cdot c} \cdot \left(\sum_n SL_{F_n} \right) \quad$ and $\quad A_D = \frac{e}{p \cdot c} \cdot \left(\sum_n SL_{D_n} \right)$

=>
$$\begin{cases} \Delta\alpha_p = -\frac{1}{2 \cdot C} \cdot \left(A_F \cdot D_{SF}^3 + A_D \cdot D_{SD}^3 \right) \\ \Delta\xi_x = \frac{1}{4 \cdot \pi} \cdot \left(A_F \cdot D_{SF} \cdot \beta_{xSF} + A_D \cdot D_{SD} \cdot \beta_{xSD} \right) \\ \Delta\xi_y = \frac{-1}{4 \cdot \pi} \cdot \left(A_F \cdot D_{SF} \cdot \beta_{ySF} + A_D \cdot D_{SD} \cdot \beta_{ySD} \right) \end{cases}$$

$$\begin{pmatrix} \Delta\xi_x \\ \Delta\xi_y \end{pmatrix} = \frac{1}{4 \cdot \pi} \cdot \begin{pmatrix} D_{SF} \cdot \beta_{xSF} & D_{SD} \cdot \beta_{xSD} \\ -D_{SF} \cdot \beta_{ySF} & -D_{SD} \cdot \beta_{ySD} \end{pmatrix} \cdot \begin{pmatrix} A_F \\ A_D \end{pmatrix}$$

=> $$\begin{pmatrix} A_F \\ A_D \end{pmatrix} = 4 \cdot \pi \cdot \begin{pmatrix} D_{SF} \cdot \beta_{xSF} & D_{SD} \cdot \beta_{xSD} \\ -D_{SF} \cdot \beta_{ySF} & -D_{SD} \cdot \beta_{ySD} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \Delta\xi_x \\ \Delta\xi_y \end{pmatrix}$$ => $$\Delta\alpha_p = -\frac{2 \cdot \pi}{C} \cdot \left(D_{SF}^3 \quad D_{SD}^3 \right) \cdot \begin{pmatrix} D_{SF} \cdot \beta_{xSF} & D_{SD} \cdot \beta_{xSD} \\ -D_{SF} \cdot \beta_{ySF} & -D_{SD} \cdot \beta_{ySD} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \Delta\xi_x \\ \Delta\xi_y \end{pmatrix}$$

$$K_\xi := -\frac{2 \cdot \pi}{C} \cdot \left(D_{SF}^3 \quad D_{SD}^3 \right) \cdot \begin{pmatrix} D_{SF} \cdot \beta_{xSF} & D_{SD} \cdot \beta_{xSD} \\ -D_{SF} \cdot \beta_{ySF} & -D_{SD} \cdot \beta_{ySD} \end{pmatrix}^{-1} = \left(-3.751 \times 10^{-3} \quad 8.033 \times 10^{-4} \right)$$

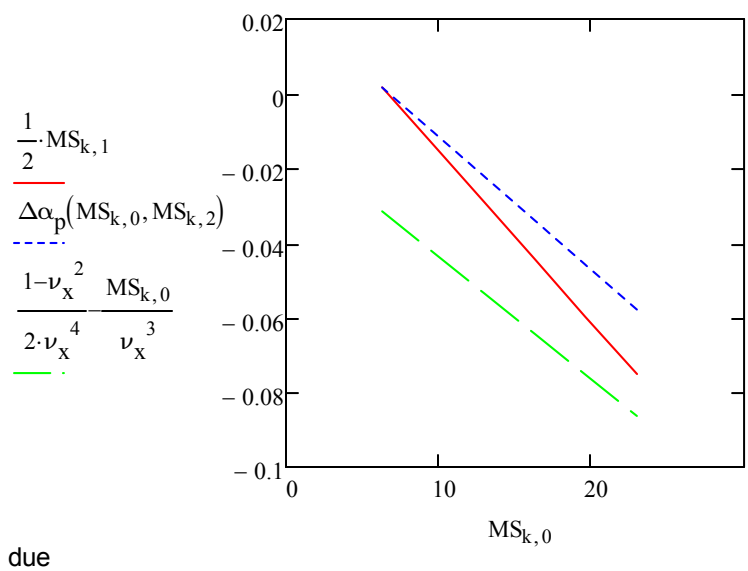
$\kappa s := 1.$

$$\Delta\alpha_p(\Delta\xi_x, \Delta\xi_y) := \kappa s \cdot \left[0.02 + \left(K_\xi^T \right)_0 \cdot \Delta\xi_x + \left(K_\xi^T \right)_1 \cdot \Delta\xi_y \right]$$

$$\frac{\Delta s}{C} = \left(\alpha - \frac{1}{\gamma^2} \right) \cdot \frac{\Delta p}{p} + \left(\alpha_p + \frac{3}{2} \cdot \frac{\beta^2}{\gamma^2} \right) \cdot \left(\frac{\Delta p}{p} \right)^2$$

$\eta_{1opt} := -0.041$

$\sigma p_{tr} := 0.002$ - rms mom. spread at transition



Non-linear time

$$d\gamma dt_b := \frac{\gamma_{n_{tr}+1} - \gamma_{n_{tr}}}{t_{n_{tr}+1} - t_{n_{tr}}} = 488.568$$

$$\tau_{bNL} := \frac{\left(\gamma_{n_{tr}} \right)^3}{2 \cdot d\gamma dt_b} \cdot \left| \eta_{1opt} \right| \cdot \sigma p_{tr}$$

$\tau_{bNL} \cdot 10^6 = 13.714$ μs

$$\frac{\varepsilon x_{n_{tr}}}{2 \cdot C} \cdot \sum_{j=0}^{N-2} \frac{\left(s_{j+1} - s_j \right)}{\beta x_j} = 4.12 \times 10^{-8}$$

- rel. orbit lengthening due to betatron motion

$$\left| \eta_{1opt} \right| \cdot \sigma p_{tr}^2 = 1.64 \times 10^{-7}$$

- rel. orbit lengthening due to 2-nd order slip-factor

$$\sigma p_{tr} := \frac{1}{3^{\frac{1}{6}} \Gamma\left(\frac{2}{3} \right)} \cdot \sqrt{\frac{2 \cdot \varepsilon_{LB\sigma}}{3 \cdot m_p}} \cdot \left[\frac{\left(2 \cdot \pi \cdot f0_{n_{tr}} \right)^2 \cdot q_b \cdot \left| \cos\left(\phi 0a_{n_{tr}} \right) \right|}{\pi \cdot \sqrt{d\gamma dt_b} \cdot \left(\beta_{n_{tr}} \right)^5 \cdot \gamma_{n_{tr}}} \cdot \frac{V0_{n_{tr}}}{m_p} \right]^{\frac{1}{3}} = 1.378 \times 10^{-3}$$

- rms momentum spread at transition computed for zero intensity

Recycler / Main Injector

Number of slip-stacked batches: $n_{injMI} := 12$

Number of other Booster cycles in the MI cycle: $n_{others} := 12$

$E_R := E_{max}$ $\quad pc_R := P_{max}$ $\quad T_{MI} := \frac{n_{injMI} + n_{others}}{f_{ramp}}$

Injection energy: $E_R = 8 \times 10^9$ eV

Top energy: $E_{MI} := 120 \cdot 10^9$ eV

$pc_{MI} := \sqrt{\left(E_{MI} + m_p\right)^2 - m_p^{\,2}}$

Injection Momentum: $pc_R = 8.889 \times 10^9$ eV

Top momentum: $pc_{MI} \cdot 10^{-9} = 120.935$ eV

Main Injector cycle duration: $T_{MI} = 1.2$ s

Number of particles at injection: $N_{be} \cdot n_{injMI} = 7.724 \times 10^{13}$

Recycler efficiency: $\kappa_{MI} := 0.97$

Number of particles at extraction: $N_{MI} = 7.493 \times 10^{13}$

$N_{MI} := N_{be} \cdot n_{injMI} \cdot \kappa_{MI}$ $\quad P_{MI} := \frac{e_{SI} \cdot N_{MI} \cdot E_{MI}}{T_{MI}}$

Beam power: $P_{MI} \cdot 10^{-6} = 1.2$ MW

Longitudinal 97% emittance of single slip-stacked bunch: $\varepsilon_{Lmax} = 0.1$ eV s

$C_{MI} := 7 \cdot C$

MI circumference: $C_{MI} \cdot 0.01 = 3319.415$ m

Transverse emittance delution at Booster-Recycler transfer: $\kappa_{\varepsilon 1} := 10\%$

Transverse emittance delution at Recycler-MI transfer: $\kappa_{\varepsilon 2} := 10\%$

$\varepsilon_{nxMI95} := \left(1 + \kappa_{\varepsilon 1}\right) \cdot \left(1 + \kappa_{\varepsilon 1}\right) \cdot \varepsilon_{nx95}$

$\varepsilon_{nyMI95} := \left(1 + \kappa_{\varepsilon 1}\right) \cdot \left(1 + \kappa_{\varepsilon 1}\right) \cdot \varepsilon_{ny95}$

Hor MI norm. emittance: $\varepsilon_{nxMI95} \cdot 10^4 = 19.36$ mm mrad

Vert MI norm. emittance: $\varepsilon_{nxMI95} \cdot 10^4 = 19.36$ mm mrad

MI cycle time: 0.7 s at 60 GeV
1.2 s at 120 GeV

Number of Booster cycles in cycle time:
$0.7 \cdot 20 = 14$ at 60 GeV
$1.2 \cdot 20 = 24$ at 120 GeV

Booster power for 8 GeV program:

$P_B \cdot 10^{-3} \cdot \frac{14 - 12}{14} = 23.572$ kW at 60 GeV

$P_B \cdot 10^{-3} \cdot \frac{12}{24} = 82.502$ kW at 120 GeV

Recycler

Harmonic number: $q_R := 588$ $\quad f0_R := \frac{c \cdot \beta_{N_{turn}}}{C_{MI}}$ $\quad f_{RF_R} := q_R \cdot f0_R$

Momentum compaction: $\alpha_R := .002508$

Slip factor: $\eta_R = -8.511 \times 10^{-3}$ $\quad \eta_R := \alpha_R - \frac{1}{\left(\gamma_{N_{turn}}\right)^2}$ $\quad K_R := \frac{\alpha_R \cdot \left(\gamma_{N_{turn}}\right)^2 - 1}{\left(\gamma_{N_{turn}}\right)^2 - 1}$

Revolution frequency: $f0_R \cdot 10^{-3} = 89.816$ kHz

RF frequency: $f_{RF_R} \cdot 10^{-6} = 52.812$ MHz $\quad \Delta f_{slip} := f_{ramp} \cdot q_b$ $\quad \Delta p_{slip} := \frac{\Delta f_{slip}}{|\eta_R| \cdot f_{RF_R}}$

RF frequency difference between slip-stacked trains: $\Delta f_{slip} = 1.68 \times 10^3$ Hz

Rel momentum difference between slip-stacked trains: $\Delta p_{slip} = 3.738 \times 10^{-3}$ $\quad \Delta E_{slip} := \frac{pc_R^{\,2}}{E_R + m_p} \cdot \Delta p_{slip} = 3.304 \times 10^7$ $\quad \varepsilon_{LR\sigma} := \varepsilon_{LB\sigma} \cdot \left(1 + \kappa_{Ltr}\right)$

<u>Beam and RF parameters before bunch coalescing (single not-slip-stacked bunch)</u>

RF voltage: $V0_R := 140 \cdot 10^3$ V $\quad \nu_{sR} := \sqrt{\frac{V0_R \cdot q_R \cdot |K_R|}{2 \cdot \pi \cdot m_p \cdot \gamma_{N_{turn}}}}$ $\quad \Delta p_p_{maxR} := \frac{2 \cdot \nu_{sR}}{q_R \cdot |\eta_R|}$

Sinchrotron tune: $\nu_{sR} = 3.552 \times 10^{-3}$

RF bucket hight: $\Delta p_p_{maxR} = 1.419 \times 10^{-3}$ $\quad \varepsilon_{LaccepR} := \frac{8}{\pi} \cdot \frac{m_p \cdot C_{MI}}{c \cdot q_R \cdot \beta_{N_{turn}}} \cdot \sqrt{\frac{V0_R \cdot \gamma_{N_{turn}}}{2 \cdot \pi \cdot m_p \cdot q_R \cdot |K_R|}}$

RF bucket area: $\varepsilon_{LaccepR} = 0.303$ eV s

Rms longitudinal emittance: $\varepsilon_{LR\sigma} = 5.504 \times 10^{-3}$ eV s $\quad \sigma_{\varphi R} := q_R \cdot \sqrt{\frac{2 \cdot \pi \cdot \varepsilon_{LR\sigma} \cdot c \cdot |\eta_R|}{m_p \cdot C_{MI} \cdot \beta_{N_{turn}} \cdot \gamma_{N_{turn}} \cdot \nu_{sR}}}$

Rms bunch length: $\sigma_{\varphi R} = 30.914 \cdot deg$

Rms rel. momentum spread: $\sigma_{pR} = 3.829 \times 10^{-4}$ $\quad \sigma_{pR} := \sqrt{\frac{2 \cdot \pi \cdot \varepsilon_{LR\sigma} \cdot c \cdot \nu_{sR}}{m_p \cdot C_{MI} \cdot \beta_{N_{turn}} \cdot \gamma_{N_{turn}} \cdot |\eta_R|}}$ $\quad B_R := \frac{\sqrt{2 \cdot \pi}}{\sigma_{\varphi R}}$

Bunching factor: $B_R = 4.646$

97% bunch length: $\tau_{bR} := 4.1 \cdot 10^{-9}$ s $\quad \varphi_{bR} := 2 \cdot \pi \cdot \tau_{bR} \cdot f_{RF_R}$ $\quad F\varepsilon(\varphi) := \int_0^{\varphi} \sqrt{\sin(\varphi)^2 - \sin(\psi)^2}\, d\psi$

$\varphi_{bR} = 1.36$ rad

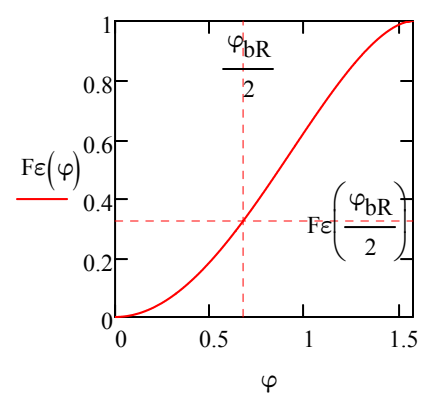
97% bunch emittance: $\varepsilon_{LR97} = 0.1$ eV s $\quad \varepsilon_{LR97} := \varepsilon_{LaccepR} \cdot F\varepsilon\left(0.5 \cdot \varphi_{bR}\right)$ $\quad \Delta p_p_{R97} := \Delta p_p_{maxR} \cdot \sqrt{\frac{1 - \cos\left(\varphi_{bR}\right)}{2}}$

97% bunch height: $\Delta p_p_{R97} = 8.928 \times 10^{-4}$ $\quad \Delta E_{R97} := m_p \cdot \gamma_{N_{turn}} \cdot \left(\beta_{N_{turn}}\right)^2 \cdot \Delta p_p_{R97}$

97% bunch height in energy: $\Delta E_{R97} \cdot 10^{-6} = 7.892$ MeV

$\varepsilon_{xR} := \varepsilon x_{N_{turn}-1} \cdot \left(1 + \kappa_{\varepsilon 1}\right)$

$\varepsilon_{yR} := \varepsilon y_{N_{turn}-1} \cdot \left(1 + \kappa_{\varepsilon 1}\right)$



95% norm. hor. emittance: $6 \cdot \varepsilon_{xR} \cdot \beta_{N_{turn}} \cdot \gamma_{N_{turn}} \cdot 10^4 = 33$ mm mrad

95% norm. vert. emittance: $6 \cdot \varepsilon_{yR} \cdot \beta_{N_{turn}} \cdot \gamma_{N_{turn}} \cdot 10^4 = 33$ mm mrad

$\varepsilon_{xR} \cdot 10^4 = 0.581$ mm mrad

$\varepsilon_{yR} \cdot 10^4 = 0.581$ mm mrad

Ring Twiss parameters

MR := RecyclerTwiss.dat

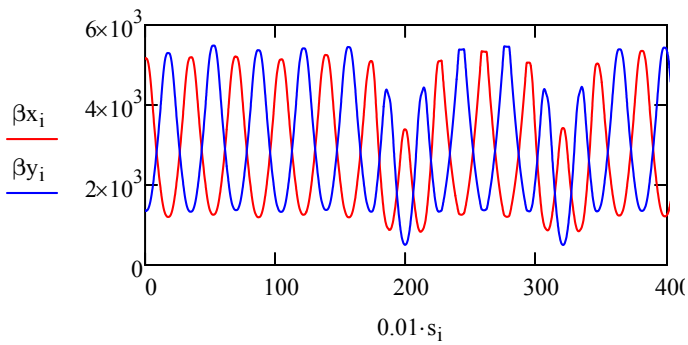
$i := 0 .. rows(MR) - 1$ $\quad rows(MR) = 1.549 \times 10^4$

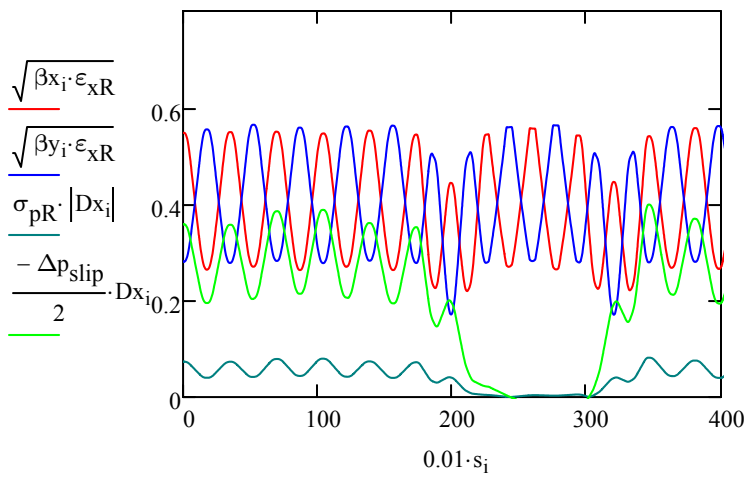
$s_i := MR_{i,2}$ $\quad \beta x_i := MR_{i,3}$ $\quad \beta y_i := MR_{i,5}$ $\quad Dx_i := MR_{i,7}$ $\quad \nu x_i := MR_{i,11}$ $\quad \nu y_i := MR_{i,12}$ $\quad \alpha x_i := MR_{i,4}$

$\alpha y_i := MR_{i,6}$ $\quad Dxp_i := MR_{i,8}$ $\quad \nu_{xR} := \nu x_{rows(MR)-1}$ $\quad \nu_{yR} := \nu y_{rows(MR)-1}$ $\quad \nu_{xR} = 25.429$ $\quad \nu_{yR} = 24.416$

$$\delta\nu_{ySC_R} := \frac{r_p\cdot(7\cdot N_{be})\cdot B_R}{2\cdot\pi\cdot(\beta_{N_{turn}})^2\cdot(\gamma_{N_{turn}})^3\cdot C_{MI}}\cdot\sum_{j=0}^{rows(MR)-2}\frac{\beta y_j\cdot(s_{j+1}-s_j)}{\sqrt{\beta y_j\cdot\varepsilon_{yR}}\cdot\left[\sqrt{\beta x_j\cdot\varepsilon_{xR}+(\sigma_{pR}\cdot Dx_j)^2}+\sqrt{\beta y_j\cdot\varepsilon_{yR}}\right]}$$

$$\delta\nu_{xSC_R} := \frac{r_p\cdot(7\cdot N_{be})\cdot B_R}{2\cdot\pi\cdot(\beta_{N_{turn}})^2\cdot(\gamma_{N_{turn}})^3\cdot C_{MI}}\cdot\sum_{j=0}^{rows(MR)-2}\frac{\beta x_j\cdot(s_{j+1}-s_j)}{\sqrt{\beta x_j\cdot\varepsilon_{xR}+(\sigma_{pR}\cdot Dx_j)^2}\cdot\left[\sqrt{\beta x_j\cdot\varepsilon_{xR}+(\sigma_{pR}\cdot Dx_j)^2}+\sqrt{\beta y_j\cdot\varepsilon_{yR}}\right]}$$





Maximum H. SC tune shift for Gaussian tr. distribution: $\delta\nu_{xSC_R} = 0.05$

Maximum V. SC tune shift for Gaussian tr. distribution: $\delta\nu_{ySC_R} = 0.052$

Maximum H. SC tune shift for KV tr. distribution: $3^{-1}\cdot\delta\nu_{xSC_R} = 0.017$

Maximum V. SC tune shift for KV tr. distribution: $3^{-1}\cdot\delta\nu_{ySC_R} = 0.017$

<u>Beam and RF parameters after bunch coalescing</u>

RF voltage: $V0_{Rc} := 1.2\cdot 10^6$ V

$$\nu_{sRc} := \sqrt{\frac{V0_{Rc}\cdot q_R\cdot|K_R|}{2\cdot\pi\cdot m_p\cdot\gamma_{N_{turn}}}} \qquad \Delta p_p_{maxRc} := \frac{2\cdot\nu_{sRc}}{q_R\cdot|\eta_R|}$$

Sinchrotron tune: $\nu_{sRc} = 0.01$

RF bucket hight: $\Delta p_p_{maxRc} = 4.156 \times 10^{-3}$

$$\varepsilon_{LaccepRc} := \frac{8}{\pi}\cdot\frac{m_p\cdot C_{MI}}{c\cdot q_R\cdot\beta_{N_{turn}}}\cdot\sqrt{\frac{V0_{Rc}\cdot\gamma_{N_{turn}}}{2\cdot\pi\cdot m_p\cdot q_R\cdot|K_R|}}$$

$\Delta p_p_{maxRc}\cdot pc_R\cdot 10^{-6} = 36.939$ MeV

RF bucket area: $\varepsilon_{LaccepRc} = 0.886$ eV s

Rms longitudinal emittance: $\varepsilon_{LR\sigma} = 5.504 \times 10^{-3}$ eV s

$$\sigma_{\varphi R} := q_R\cdot\sqrt{\frac{2\cdot\pi\cdot\varepsilon_{LR\sigma}\cdot c\cdot|\eta_R|}{m_p\cdot C_{MI}\cdot\beta_{N_{turn}}\cdot\gamma_{N_{turn}}\cdot\nu_{sR}}}$$

Rms bunch length: $\sigma_{\varphi R} = 30.914\cdot deg$

$$\sigma_{pR} := \sqrt{\frac{2\cdot\pi\cdot\varepsilon_{LR\sigma}\cdot c\cdot\nu_{sR}}{m_p\cdot C_{MI}\cdot\beta_{N_{turn}}\cdot\gamma_{N_{turn}}\cdot|\eta_R|}} \qquad B_R := \frac{\sqrt{2\cdot\pi}}{\sigma_{\varphi R}}$$

Rms rel. momentum spread: $\sigma_{pR} = 3.829 \times 10^{-4}$

Bunching factor: $B_R = 4.646$

Main Injector

Ramp suggested for PIP-II

$\Delta T_r := (0.01\ \ 0.01\ \ 0.01\ \ 0.073\ \ 0.297\ \ 0.15\ \ 0.05\ \ 0.01\ \ 0.11\ \ 0.17\ \ 0.20\ \ 0.03\ \ 0.04\ \ 0.04\ \ .01)^T$ $\quad Nk := \mathrm{rows}(\Delta T_r) - 1$

$P0_r := (8.888\ \ 8.888\ \ 8.99\ \ 9.56\ \ 22.0\ \ 86\ \ 116.1\ \ 120.888\ \ 120.888\ \ 104\ \ 60.0\ \ 11\ \ 6.4\ \ 7.799\ \ 8.888)^T$

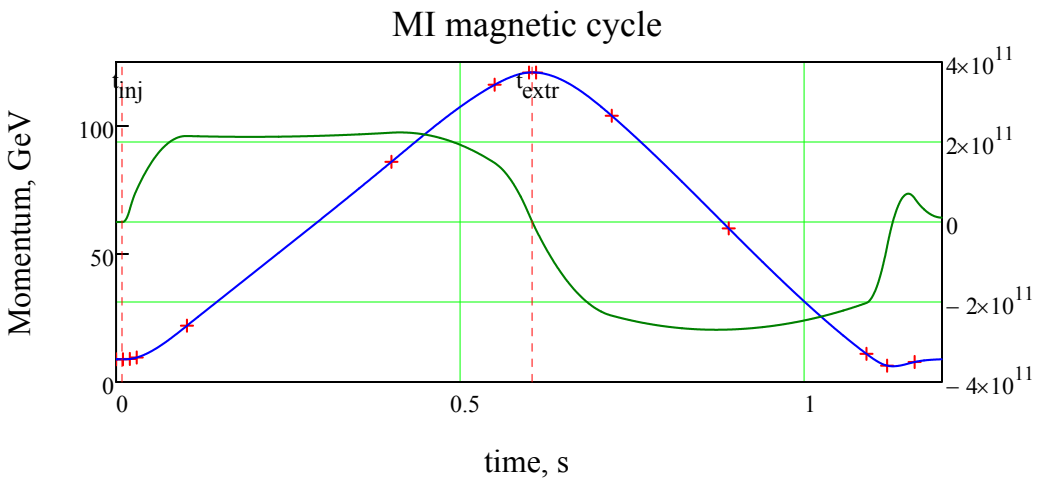
$k := 0 .. Nk - 1 \quad T_{r_0} := 0 \quad T_{r_{k+1}} := T_{r_k} + \Delta T_{r_k} \quad N_{rMI} := 2048 \quad ir := 0 .. N_{rMI} \quad T_{r_{Nk}} = 1.2$

$t_{ir} := \frac{ir}{N_{rMI}} \cdot T_{r_{Nk}} \quad vt := \mathrm{lspline}(T_r, P0_r) \quad pc_{ir} := 10^9 \cdot \mathrm{interp}(vt, T_r, P0_r, t_{ir}) \quad ir := 0 .. N_{rMI} - 1 \quad dPdt_{ir} := \frac{pc_{ir+1} - pc_{ir}}{t_{ir+1} - t_{ir}}$

$t_{inj} := 0.01 \quad t_{extr} := 0.605 \quad ir_1 := \mathrm{floor}\left[N_{rMI} \cdot \frac{t_{inj}}{(T_{r_{Nk}})}\right] \quad ir_2 := \mathrm{floor}\left[N_{rMI} \cdot \frac{t_{extr}}{(T_{r_{Nk}})}\right]$

$ir_1 = 17$

$ir_2 = 1032$



$ir := 0 .. ir_2$

$E_{mi_{ir}} := \sqrt{(pc_{ir})^2 + m_p^2} - m_p$

$E_{mi_{ir_1}} \cdot 10^{-9} = 7.999$ GeV

$E_{mi_{ir_2}} \cdot 10^{-9} = 120$ GeV

$\gamma_{mi_{ir}} := 1 + \frac{E_{mi_{ir}}}{m_p}$

Harmonic number: $q_{mi} := 588$

Slip factor: $\alpha_{mi} := .00214 \quad \gamma_{tMI} := \frac{1}{\sqrt{\alpha_{mi}}} \quad \beta_{mi_{ir}} := \sqrt{1 - \frac{1}{(\gamma_{mi_{ir}})^2}} \quad f0_{mi_{ir}} := \frac{c \cdot \beta_{mi_{ir}}}{C_{MI}}$

Transition gamma: $\gamma_{tMI} = 21.617$

$f_{RFmi_{ir}} := q_{mi} \cdot f0_{mi_{ir}}$

RF frequency at injection: $f_{RFmi_{ir_1}} \cdot 10^{-6} = 52.812$ MHz

$K_{mi_{ir}} := \frac{\alpha_{mi} \cdot (\gamma_{mi_{ir}})^2 - 1}{(\gamma_{mi_{ir}})^2 - 1} \quad \eta_{mi_{ir}} := \alpha_{mi} - \frac{1}{(\gamma_{mi_{ir}})^2}$

RF frequency at extraction: $f_{RFmi_{ir_2}} \cdot 10^{-6} = 53.104$ MHz

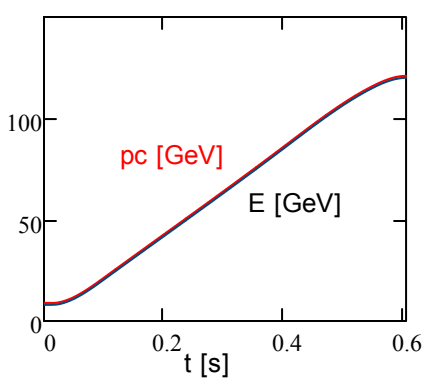
Minimum L. bucket size (total aria): $\varepsilon_{LBmi} := 0.8$ eV*s $\quad \varepsilon_{Lmi\sigma} := \frac{\varepsilon_{LBmi}}{6 \cdot \pi}$

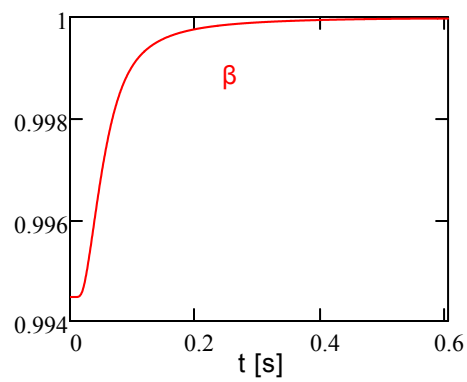
RMS L. emittance(Δx Δp): $\varepsilon_{Lmi\sigma} = 0.042$ eV*s

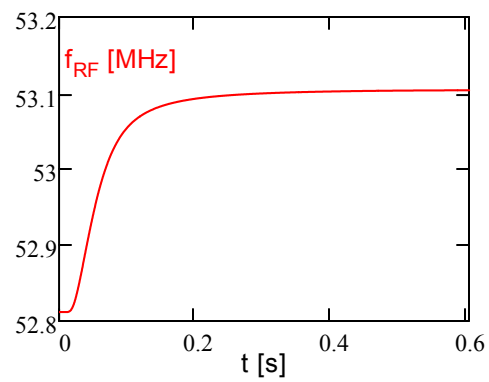
$\varepsilon_{xMI_{ir}} := \varepsilon x_{N_{turn}-1} \cdot \frac{\beta_{mi_0} \cdot \gamma_{mi_0}}{\beta_{mi_{ir}} \cdot \gamma_{mi_{ir}}} \cdot (1 + \kappa_{\varepsilon 1}) \cdot (1 + \kappa_{\varepsilon 2})$

Compare RMS L. emittance to - $\varepsilon_{LR\sigma} \cdot \frac{\Delta p_{slip}}{\sigma_{pR}} = 0.054$ eV s

$\varepsilon_{yMI_{ir}} := \varepsilon y_{N_{turn}-1} \cdot \frac{\beta_{mi_0} \cdot \gamma_{mi_0}}{\beta_{mi_{ir}} \cdot \gamma_{mi_{ir}}} \cdot (1 + \kappa_{\varepsilon 1}) \cdot (1 + \kappa_{\varepsilon 2})$







MI RF

$ir := 0 .. ir_2 - 1$ $\quad Va_{mi_{ir}} := \dfrac{E_{mi_{ir+1}} - E_{mi_{ir}}}{f0_{mi_{ir}} \cdot \left(t_{ir+1} - t_{ir}\right)}$ $\quad Va_{mi_{ir_2}} := Va_{mi_{ir_2-1}}$ $\quad ir := 0 .. ir_2$

$V_{mi} := (1050 \;\; 1050.0 \;\; 1650 \;\; 1650.0 \;\; 800 \;\; 900 \;\; 1000 \;\; 890 \;\; 790 \;\; 680 \;\; 400)^T \cdot 10^3$

$t_{mi} := \left(0.00 \;\; 0.010 \;\; 0.0200 \;\; 0.050 \;\; 0.085 \;\; 0.1200 \;\; 0.20 \;\; 0.3 \;\; 0.4 \;\; 0.50 \;\; t_{extr}\right)^T$ $\quad v_{mi} := \mathrm{pspline}\left(t_{mi}, V_{mi}\right)$

$\Delta V_{ir} := \mathrm{interp}\left(v_{mi}, t_{mi}, V_{mi}, t_{ir}\right)$ $\quad V0_{mi_{ir}} := Va_{mi_{ir}} + \Delta V_{ir}$ $\quad \phi 0_{mi_{ir}} := \mathrm{asin}\left(\dfrac{Va_{mi_{ir}}}{V0_{mi_{ir}}}\right)$ $\quad \nu_{s0mi_{ir}} := \sqrt{\dfrac{V0_{mi_{ir}} \cdot q_{mi} \cdot \left|K_{mi_{ir}}\right|}{2 \cdot \pi \cdot m_p \cdot \gamma_{mi_{ir}}}}$

$\varepsilon_{LtotMI_{ir}} := \dfrac{8}{\pi} \cdot \dfrac{m_p \cdot C_{MI}}{c \cdot q_{mi} \cdot \beta_{mi_{ir}}} \cdot \sqrt{\dfrac{V0_{mi_{ir}} \cdot \gamma_{mi_{ir}}}{2 \cdot \pi \cdot m_p \cdot q_{mi} \cdot \left|K_{mi_{ir}}\right|}} \cdot \dfrac{1 - \sin\left(\phi 0_{mi_{ir}}\right)}{\left(1 + 0.5 \cdot \sin\left(\phi 0_{mi_{ir}}\right)\right)^2}$ $\quad \nu_{smi_{ir}} := \nu_{s0mi_{ir}} \cdot \sqrt{\cos\left(\phi 0_{mi_{ir}}\right)}$

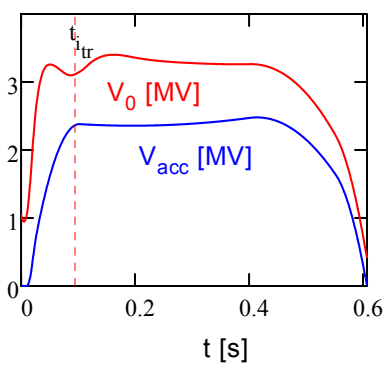
$\Delta\Phi(\phi) := 2 \cdot \pi - 1.36698 \cdot \phi - 3.3 \cdot \sqrt{\phi} + 0.208 \cdot \phi \cdot \left(\dfrac{\pi}{2} - \phi\right)$ $\quad \Delta\phi_{mi_{ir}} := \Delta\Phi\left(\phi 0_{mi_{ir}}\right)$

$\Delta p_p_{m_{ir}} := \dfrac{2 \cdot \nu_{s0mi_{ir}}}{q_{mi} \cdot \left|\eta_{mi_{ir}}\right|} \cdot \sqrt{\cos\left(\phi 0_{mi_{ir}}\right) - \left(\dfrac{\pi}{2} - \phi 0_{mi_{ir}}\right) \cdot \sin\left(\phi 0_{mi_{ir}}\right)}$ $\quad \phi 0a_{mi_{ir}} := \phi 0_{mi_{ir}} \cdot \left(K_{mi_{ir}} < 0\right) + \left(\pi - \phi 0_{mi_{ir}}\right) \cdot \left(K_{mi_{ir}} \geq 0\right)$

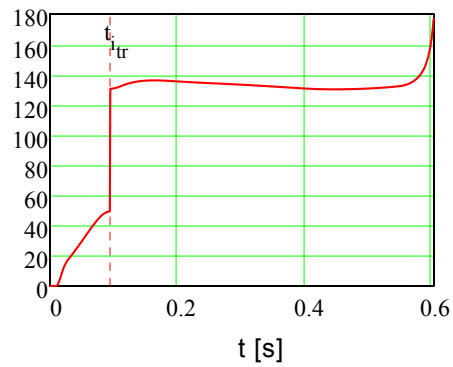
$\sigma_{\varphi mi_{ir}} := q_{mi} \cdot \sqrt{\dfrac{2 \cdot \pi \cdot \varepsilon_{Lmi\sigma} \cdot c \cdot \left|\eta_{mi_{ir}}\right|}{m_p \cdot C_{MI} \cdot \beta_{mi_{ir}} \cdot \gamma_{mi_{ir}} \cdot \nu_{smi_{ir}}} \cdot \left[1 + \kappa_{Ltr} \cdot \left(K_{mi_{ir}} \geq 0\right)\right]}$ $\quad \sigma_{pmi_{ir}} := \sqrt{\dfrac{2 \cdot \pi \cdot \varepsilon_{Lmi\sigma} \cdot c \cdot \nu_{smi_{ir}}}{m_p \cdot C_{MI} \cdot \beta_{mi_{ir}} \cdot \gamma_{mi_{ir}} \cdot \left|\eta_{mi_{ir}}\right|} \cdot \left[1 + \kappa_{Ltr} \cdot \left(K_{mi_{ir}} \geq 0\right)\right]}$

$B_{mi_{ir}} := \dfrac{\sqrt{2 \cdot \pi}}{\sigma_{\varphi mi_{ir}}}$ $\quad i_{tr} := 162$ $\quad \Sigma\nu_{smi_0} := 0$ $\quad \Sigma\nu_{smi_{ir+1}} := \Sigma\nu_{smi_{ir}} + f0_{mi_{ir}} \cdot \nu_{smi_{ir}} \cdot \left(t_{ir+1} - t_{ir}\right)$

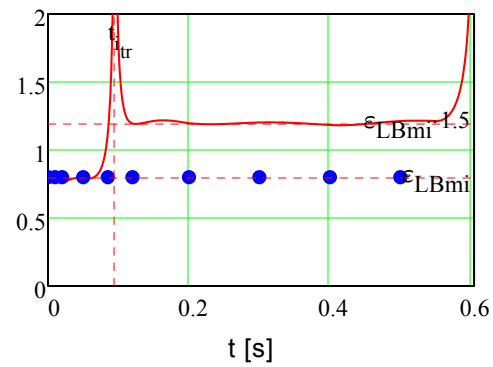
RF voltage



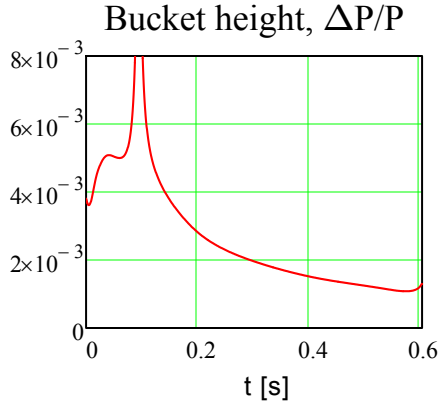
Accelerating phase, deg



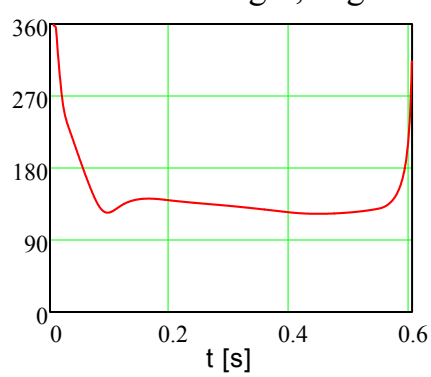
Bucket area, eV s



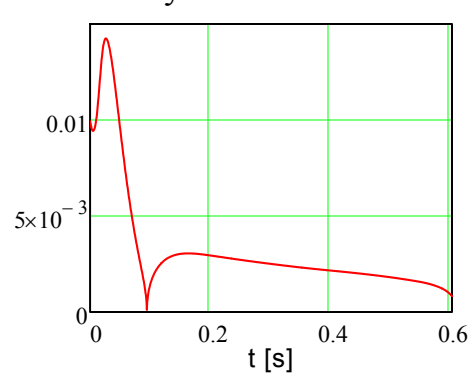
Bucket height, ΔP/P

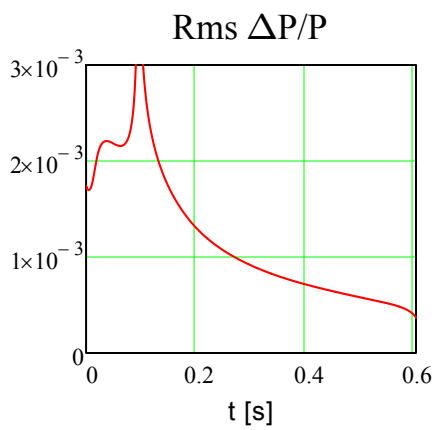


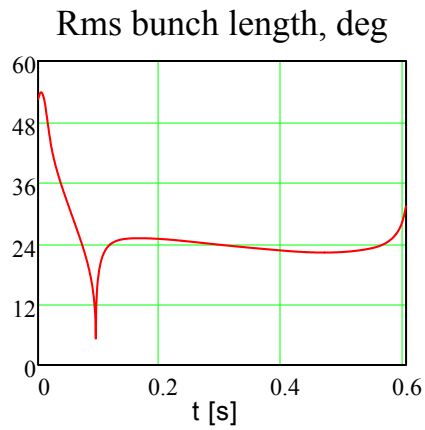
Bucket length, deg

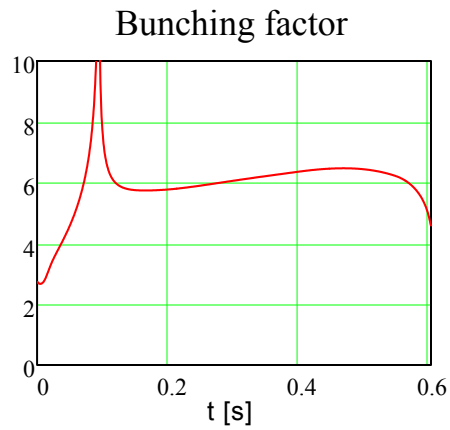


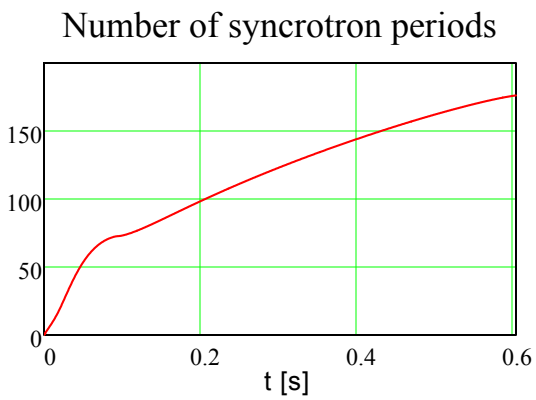
Synchrotron tune









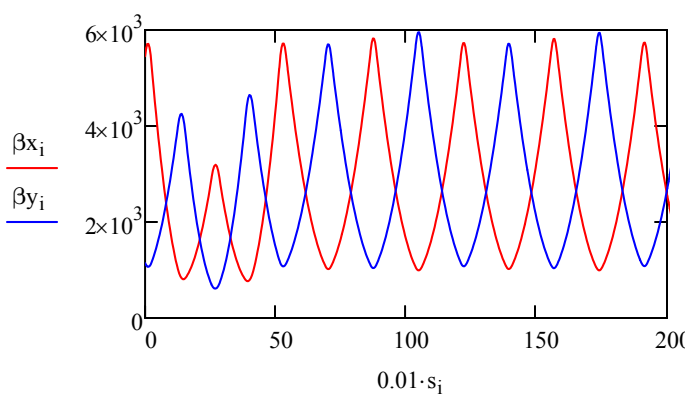


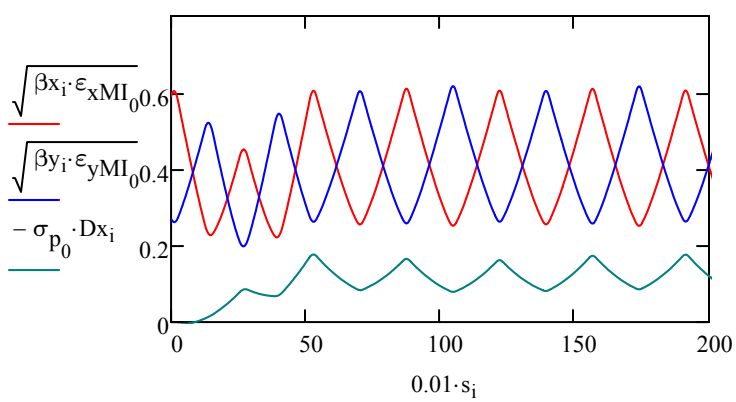
Ring Twiss parameters

$M :=$ MI_Twiss.dat $\qquad N := \mathrm{rows}(M) \qquad i := 0 .. N-1 \qquad N = 1.017 \times 10^4$

$s_i := M_{i,2} \qquad \beta x_i := M_{i,3} \qquad \beta y_i := M_{i,5} \qquad Dx_i := M_{i,7} \qquad \nu x_i := M_{i,11} \qquad \nu y_i := M_{i,12}$

$\alpha x_i := M_{i,4} \qquad \alpha y_i := M_{i,6} \qquad Dxp_i := M_{i,8} \qquad \nu_{xMI} := \nu x_{N-1} \qquad \nu_{yMI} := \nu y_{N-1} \qquad \nu_{xMI} = 26.424 \qquad \nu_{yMI} = 25.414$

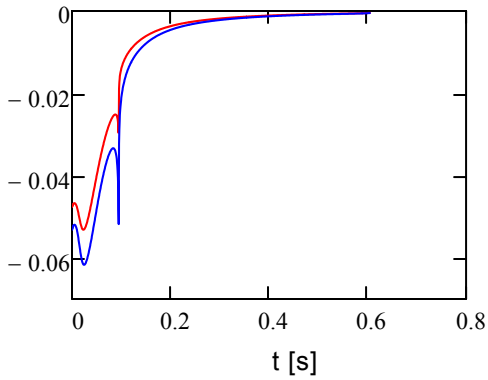




$$\delta\nu_{ySC_{ir}} := \frac{r_p \cdot (14 \cdot N_{be}) \cdot B_{mi_{ir}}}{2 \cdot \pi \cdot \left(\beta_{mi_{ir}}\right)^2 \cdot \left(\gamma_{mi_{ir}}\right)^3 \cdot C_{MI}} \cdot \sum_{j=0}^{N-2} \frac{\beta y_j \cdot \left(s_{j+1} - s_j\right)}{\sqrt{\beta y_j \cdot \varepsilon_{yMI_{ir}}} \cdot \left[\sqrt{\beta x_j \cdot \varepsilon_{xMI_{ir}} + \left(\sigma_{pmi_{ir}} \cdot Dx_j\right)^2} + \sqrt{\beta y_j \cdot \varepsilon_{yMI_{ir}}}\right]}$$

$$\delta\nu_{xSC_{ir}} := \frac{r_p \cdot (14 \cdot N_{be}) \cdot B_{mi_{ir}}}{2 \cdot \pi \cdot \left(\beta_{mi_{ir}}\right)^2 \cdot \left(\gamma_{mi_{ir}}\right)^3 \cdot C_{MI}} \cdot \sum_{j=0}^{N-2} \frac{\beta x_j \cdot \left(s_{j+1} - s_j\right)}{\sqrt{\beta x_j \cdot \varepsilon_{xMI_{ir}} + \left(\sigma_{pmi_{ir}} \cdot Dx_j\right)^2} \cdot \left[\sqrt{\beta x_j \cdot \varepsilon_{xMI_{ir}} + \left(\sigma_{pmi_{ir}} \cdot Dx_j\right)^2} + \sqrt{\beta y_j \cdot \varepsilon_{yMI_{ir}}}\right]}$$

Space charge tune shifts



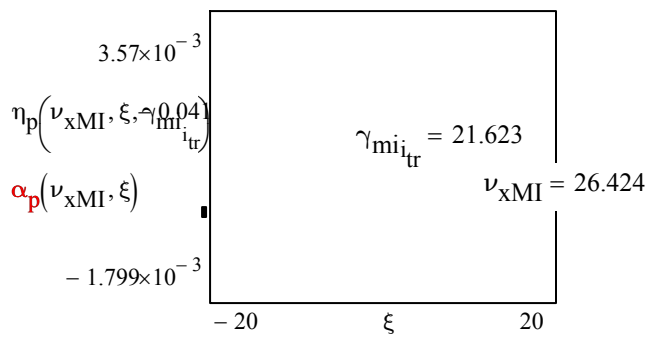
Second order slip factor and non-linear time

$\sigma p_{MItr} := 5\cdot 10^{-3}$ $\qquad \eta_{1MI} := 2.5\cdot 10^{-3}$

Non-linear time

$$\tau_{miNL} := \left(\gamma_{mi_{i_{tr}}}\right)^3 \cdot \left| \frac{\eta_{1MI}\cdot\left(t_{i_{tr}+1} - t_{i_{tr}}\right)}{2\cdot\left(\gamma_{mi_{i_{tr}+1}} - \gamma_{mi_{i_{tr}}}\right)} \right| \cdot \sigma p_{MItr}$$

$$\tau_{miNL}\cdot 10^6 = 277.678 \ \mu s$$



$$\frac{\varepsilon_{xMI_{i_{tr}}}}{2\cdot C_{MI}}\cdot\sum_{j=0}^{N-2}\frac{\left(s_{j+1} - s_j\right)}{\beta x_j} = 7.006\times 10^{-9}$$ - rel. orbit lengthening due to betatron motion

$$\left|\eta_{1MI}\right|\cdot\sigma p_{MItr}^{\ 2} = 6.25\times 10^{-8}$$ - rel. orbit lengthening due to 2-nd order slip-factor

$$d\gamma dt_{MI} := \frac{\gamma_{mi_{i_{tr}+1}} - \gamma_{mi_{i_{tr}}}}{t_{i_{tr}+1} - t_{i_{tr}}} = 227.567$$

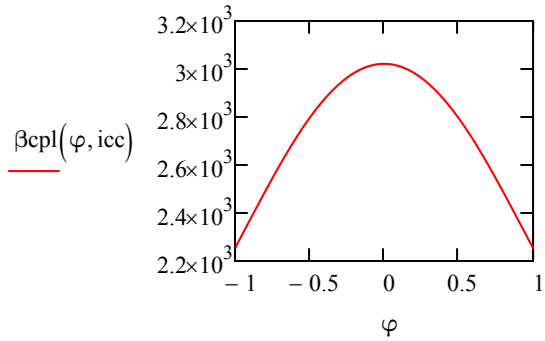
$$\sigma p_{MItr} := \frac{1}{3^{\frac{1}{6}}\,\Gamma\left(\frac{2}{3}\right)}\cdot\sqrt{\frac{2\cdot\pi\cdot\varepsilon_{Lmi\sigma}}{3\cdot m_p}}\cdot\left[\frac{\left(2\cdot\pi\cdot f0_{mi_{i_{tr}}}\right)^2\cdot q_{mi}\cdot\left|\cos\left(\phi 0_{mi_{i_{tr}}}\right)\right|}{\pi\cdot\sqrt{d\gamma dt_{MI}}\cdot\left(\beta_{mi_{i_{tr}}}\right)^5\cdot\gamma_{mi_{i_{tr}}}}\cdot\frac{V0_{mi_{i_{tr}}}}{m_p}\right]^{\frac{1}{3}} = 4.413\times 10^{-3}$$

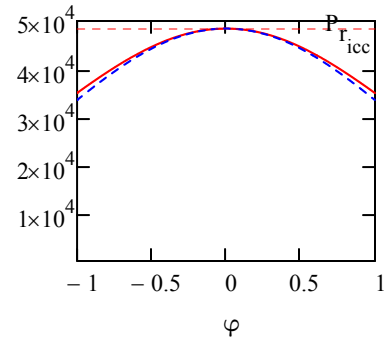
- rms momentum spread at transition computed for zero intensity

$$\beta cpl(\varphi, ic) := \sqrt{\left(1 + \frac{I_{cpl} \cdot \cos(\varphi) \cdot R_Q_{ic} \cdot Q0_{ic}}{U_{max_{ic}}}\right)^2 + \left(\frac{2 \cdot \delta f_{mf_{ic}} \cdot Q0_{ic}}{f_{c_{ic}}}\right)^2}$$

$$Preq(\varphi, \beta cpl, ic) := P_{req}\left(U_{max_{ic}}, I_{linac}, \varphi, R_Q_{ic}, Q0_{ic}, \beta cpl, f_{c_{ic}}, \delta f_{mf_{ic}}\right)$$

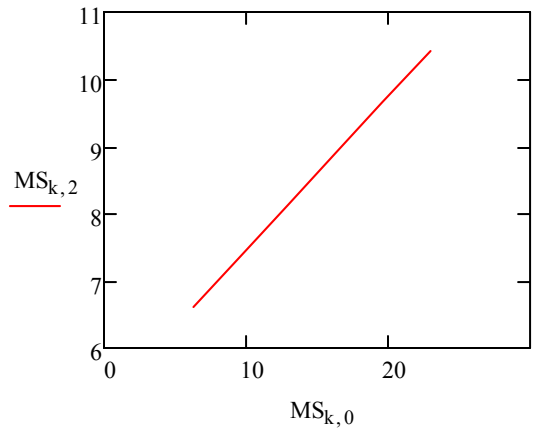
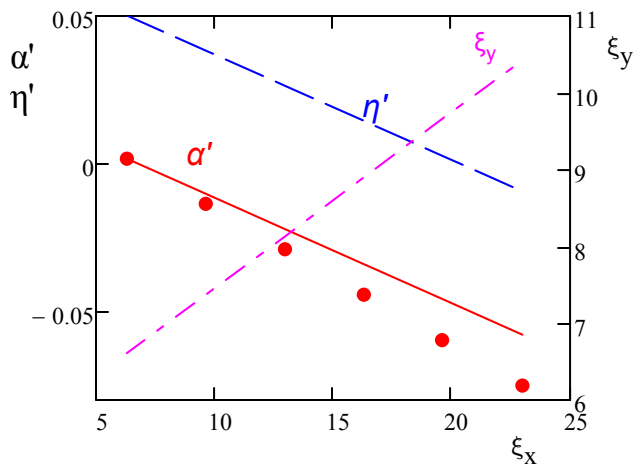
$$icc := 5$$





	$M_{21,ic}$ =		$\Delta f_{LFD_{ic}}$ =		$\frac{\Delta f_{LFD_{ic}}}{\Delta f_{c_{ic}}}$ =
	0		0		0
	0		0		0
SSR1	5	$\frac{Hz}{\left(\frac{MeV}{m}\right)^2}$	501.068	Hz	4.658
SSR2	2.8		362.723		5.636
LB650	1.42		406.469		6.466
HB650	1		351.913		5.374

$$\left(\gamma_t - 1\right)\cdot m_p = 4.191 \times 10^9$$



$opt := 0 \quad \eta_{p_{opt}} := -0.041$