PIP-II Linac Beam parameters and Intrabeam Stripping

Fundamental constants

$m_p := 938.256\cdot10^6$ eV $\quad M_p := 1.6725\cdot10^{-24}$ $\quad m_e := 0.511006\cdot10^6$ eV $\quad M_e := 9.109081\cdot10^{-28}$ $\quad e_{SI} := 1.6021\cdot10^{-19}$

$m_\mu := 105.658\cdot10^6$ eV $\quad m_\pi := 139.57\cdot10^6$ eV $\quad e_{conv} := 299.7925$ $\quad e_{SGS} := 4.802981\cdot10^{-10}$ $\quad h_P := 1.054491\cdot10^{-27}$

$c := 2.997925\cdot10^{10}$ cm/c $\quad Z_0 := 377$ Ω $\quad r_p := \frac{e_{SGS}\cdot e_{conv}}{m_p}$ $\quad r_e := \frac{e_{SGS}\cdot e_{conv}}{m_e}$ $\quad \alpha_{FS} := \frac{e_{SGS}^2}{h_P\cdot c}$ $\quad a_0 := \frac{h_P^2}{M_e\cdot e_{SGS}^2}$

$\tau_\mu := 2.197\cdot10^{-6}$ s $\quad \tau_\pi := 2.603\cdot10^{-8}$ s

Data reading from TraceWin simulations

position gam-1 rms_size(x,x',y,y',z,dp/p,z',phase,time,energie) centroid position(x,x',y,y',z,dp/p,z',phase,time,energy) (dispX,dispY,betX,betY) unit(m,rad,deg,s,MeV)

$M :=$ ...\Envelopes.txt $\quad N := \text{rows}(M) \quad i := 0..N-1 \quad N = 3.091\times10^4$

$s_i := 100\cdot M_{i,0} \quad En_i := M_{i,1}\cdot m_p \quad \sigma x_i := 100\cdot M_{i,12} \quad \theta x_i := M_{i,13} \quad \sigma y_i := 100\cdot M_{i,14} \quad \theta y_i := M_{i,15} \quad \sigma s_i := 100\cdot M_{i,16} \quad \Delta p_i := M_{i,17}$

$s_0 := 2000 \quad a := 800 \quad a_1 := 2000 \quad s_1 := 7000 \quad s_2 := 1900 \quad a_2 := 800$

$\gamma_i := M_{i,1} + 1 \quad \beta_i := \sqrt{1 - \frac{1}{(\gamma_i)^2}}$

$$\varepsilon nx(s) := 10^{-4}\cdot\left[0.225 + 0.047\cdot\frac{a\cdot(s-s_0)}{(s-s_0)^2 + a^2}\cdot\left[1 - \frac{1}{1 + \left(\frac{s-s_1}{a_1}\right)^2}\right] + 2\cdot10^{-7}\cdot s\right]$$

$$\varepsilon ny(s) := 10^{-4}\cdot\left[0.22 + 0.03\cdot\frac{a\cdot(s-s_0)}{(s-s_0)^2 + a^2} + \frac{0.012}{1 + \left(\frac{s-10^4}{3\cdot10^3}\right)^2} + 24\cdot10^{-7}\cdot s\right]$$

$$\varepsilon nL(s) := 10^{-4}\cdot\left[0.29 - 0.038\cdot\frac{a_2\cdot(s-s_2)}{(s-s_2)^2 + a_2^2} - 12\cdot10^{-7}\cdot s\right]$$

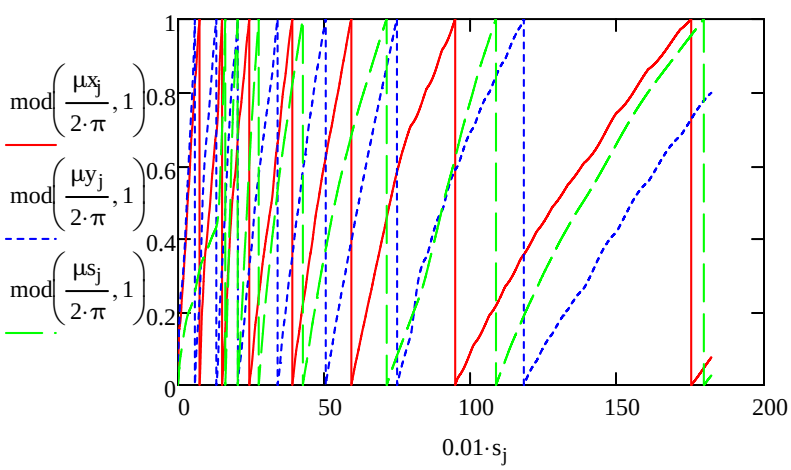
$\sigma_{\theta x_i} := \frac{\varepsilon nx(s_i)}{\beta_i\cdot\gamma_i\cdot\sigma x_i} \quad \beta x_i := \frac{(\sigma x_i)^2\cdot(\beta_i\cdot\gamma_i)}{\varepsilon nx(s_i)}$

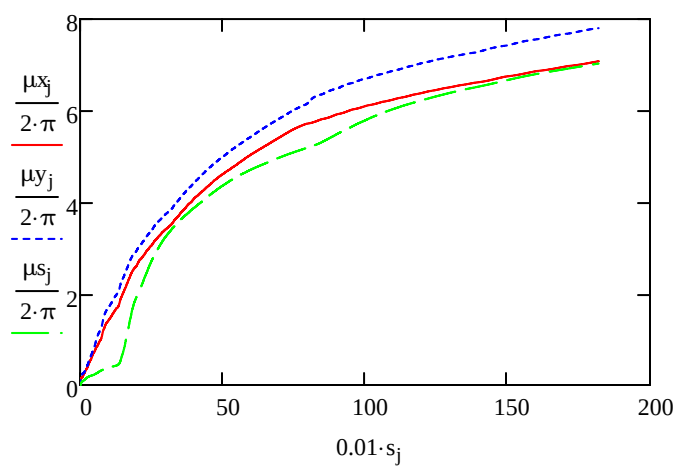
$\sigma_{\theta y_i} := \frac{\varepsilon ny(s_i)}{\beta_i\cdot\gamma_i\cdot\sigma y_i} \quad \beta y_i := \frac{(\sigma y_i)^2\cdot(\beta_i\cdot\gamma_i)}{\varepsilon ny(s_i)}$

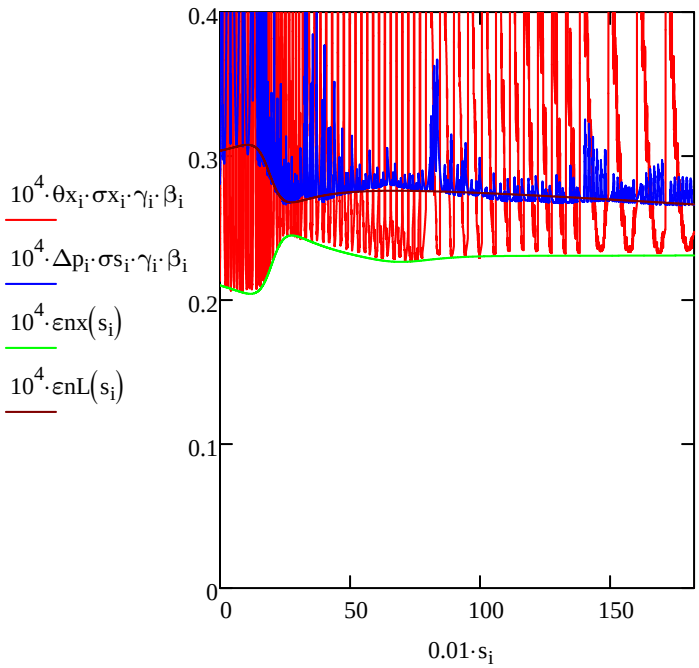
$\sigma_{p_i} := \frac{\varepsilon nL(s_i)}{\beta_i\cdot\gamma_i\cdot\sigma s_i} \quad \beta s_i := \frac{(\sigma s_i)^2\cdot(\beta_i\cdot\gamma_i)}{\varepsilon nL(s_i)}$

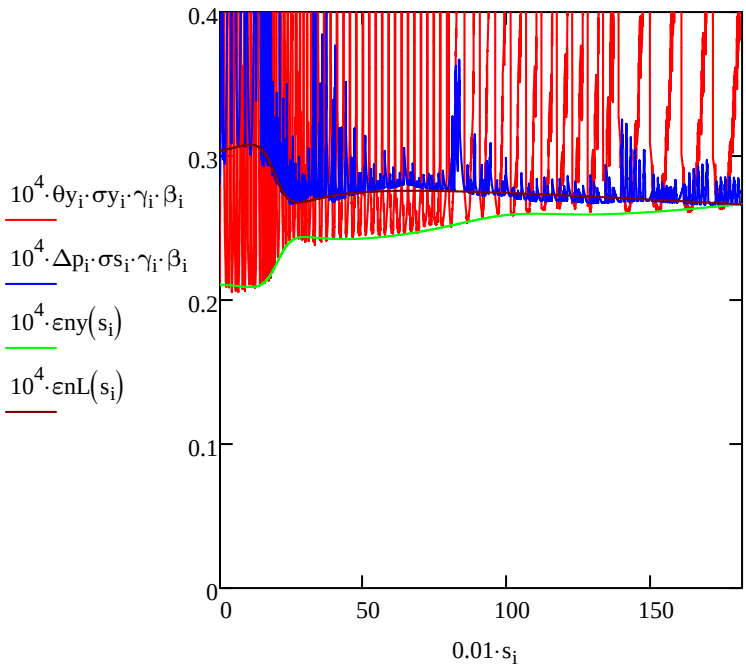
$j := 0..N-2 \quad \alpha x_j := \frac{1}{2}\cdot\frac{\beta x_j - \beta x_{j+1}}{s_{j+1} - s_j + 10^{-8}} \quad \alpha y_j := \frac{1}{2}\cdot\frac{\beta y_j - \beta y_{j+1}}{s_{j+1} - s_j + 10^{-8}} \quad \alpha s_j := \frac{1}{2}\cdot\frac{\beta s_j - \beta s_{j+1}}{s_{j+1} - s_j + 10^{-8}}$

$\mu x_0 := 0 \quad \mu y_0 := 0 \quad \mu s_0 := 0 \quad \mu x_{j+1} := \mu x_j + \frac{s_{j+1} - s_j}{\beta x_j} \quad \mu y_{j+1} := \mu y_j + \frac{s_{j+1} - s_j}{\beta y_j} \quad \mu s_{j+1} := \mu s_j + \frac{s_{j+1} - s_j}{(\gamma_j)^2\cdot\beta s_j}$

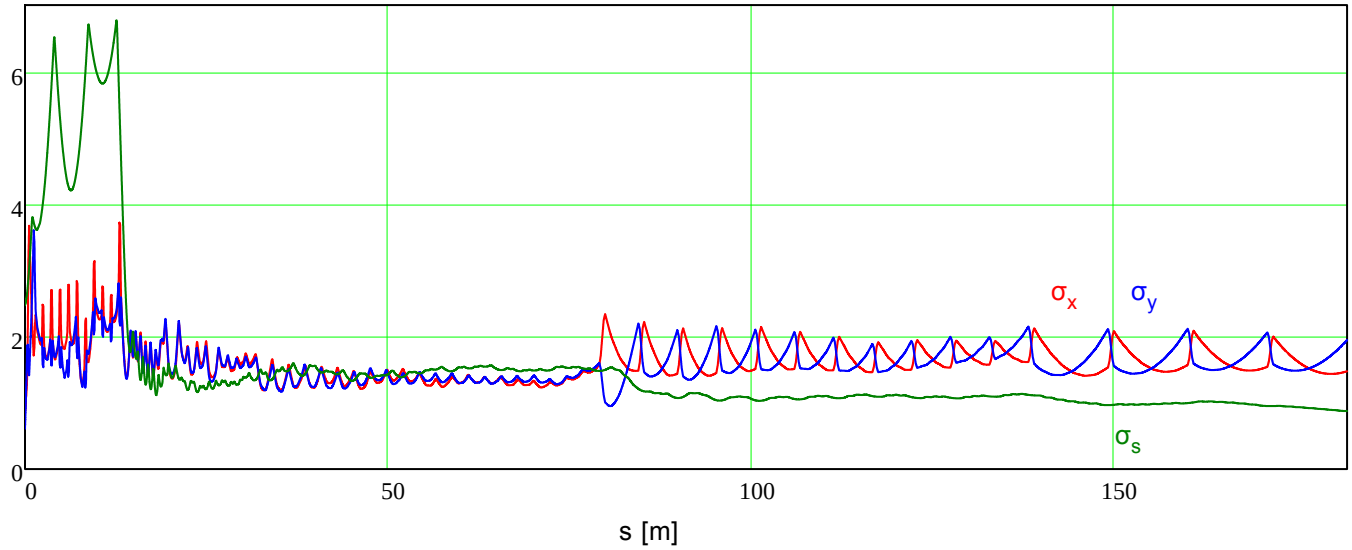




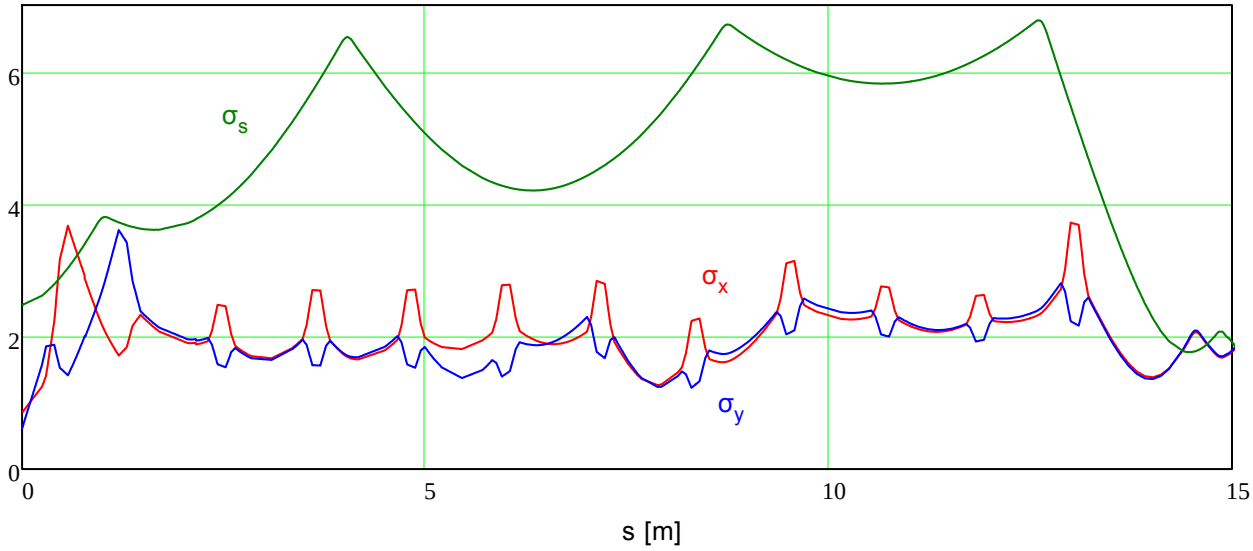




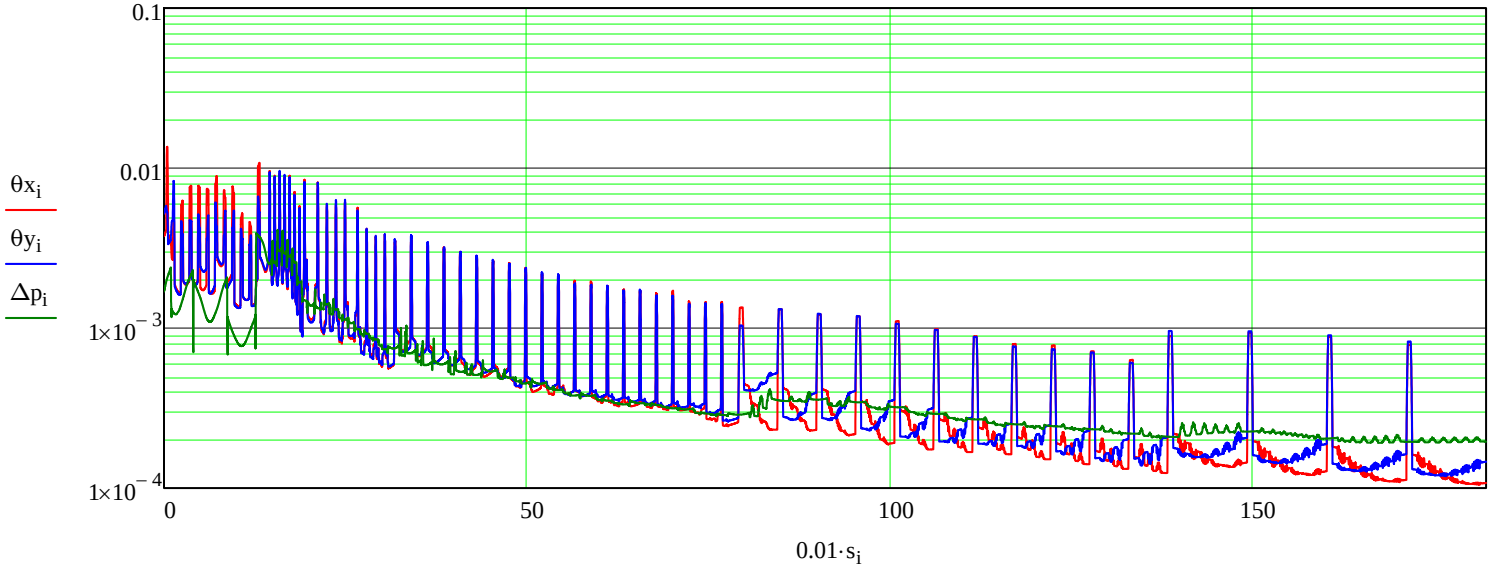
Rms beam sizes [mm]



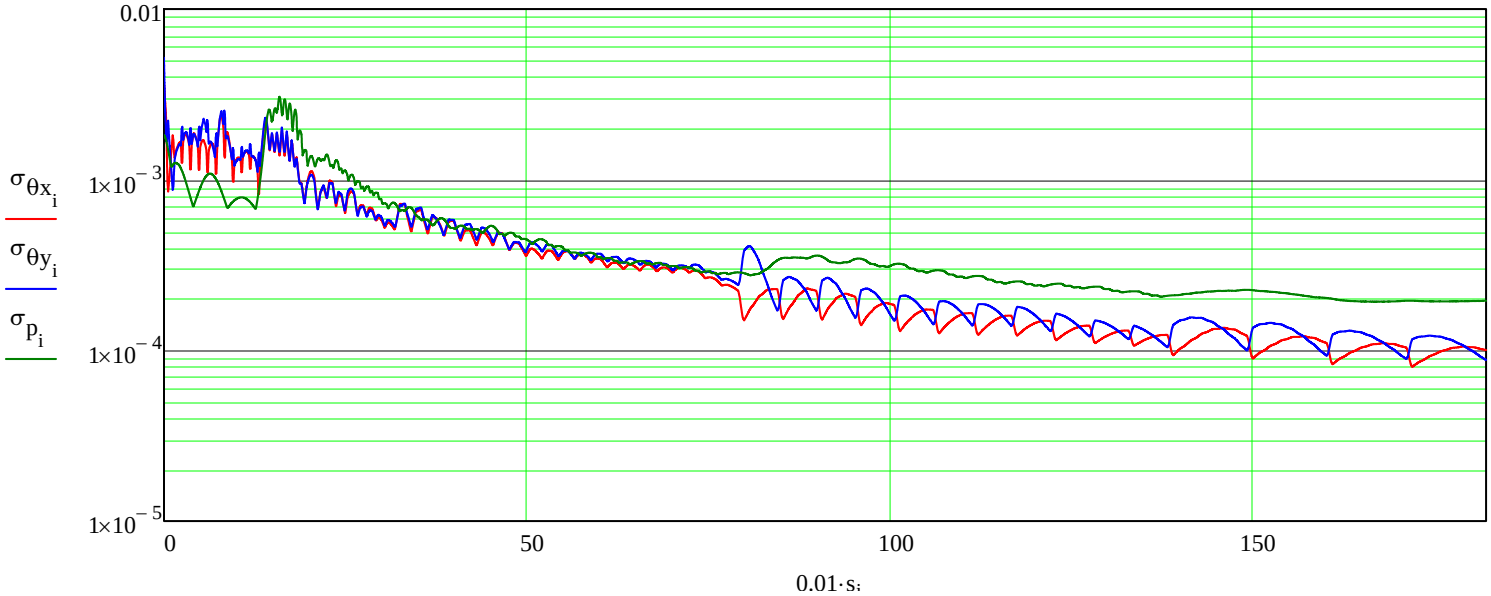
Rms beam sizes [mm]



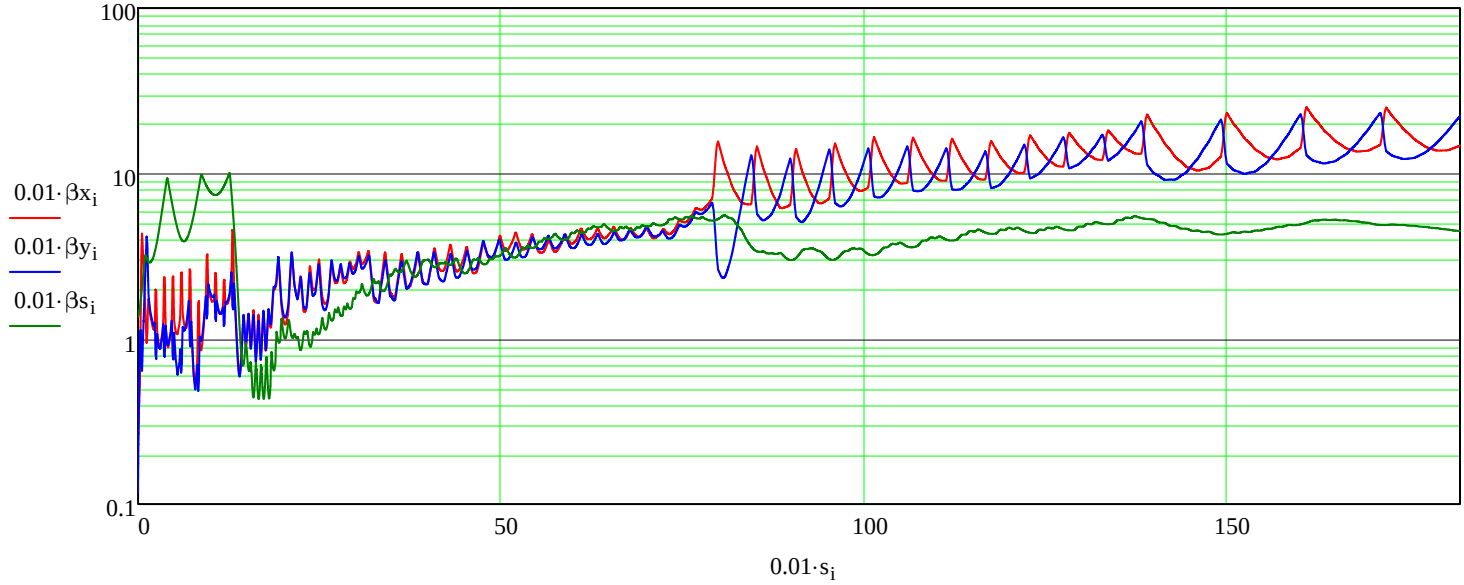
Relative rms momentum spreads

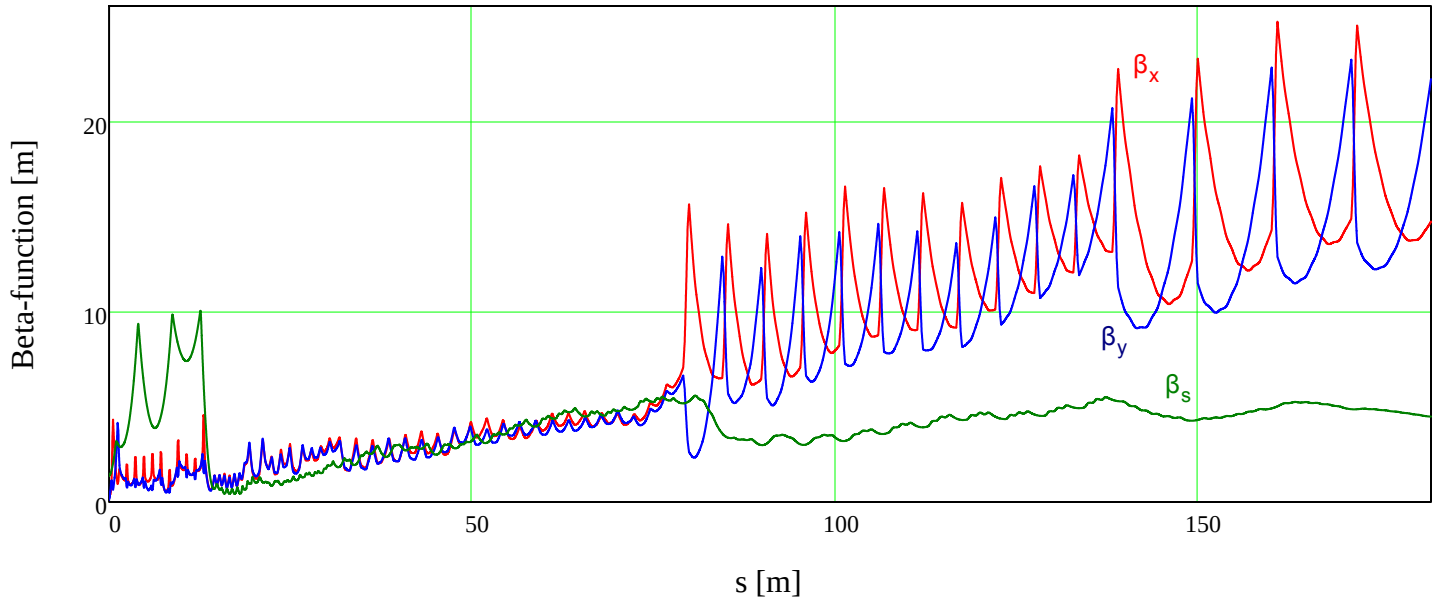


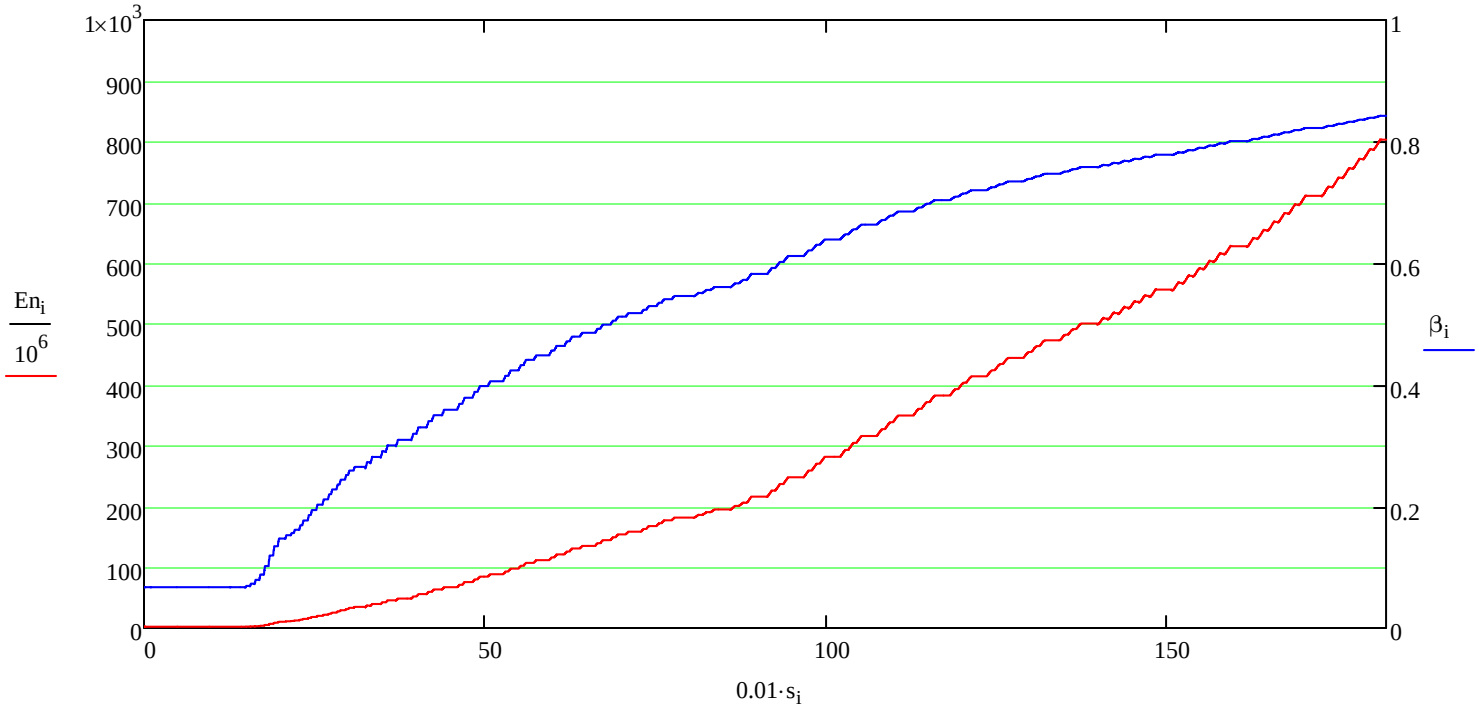
Relative local rms momentum spreads

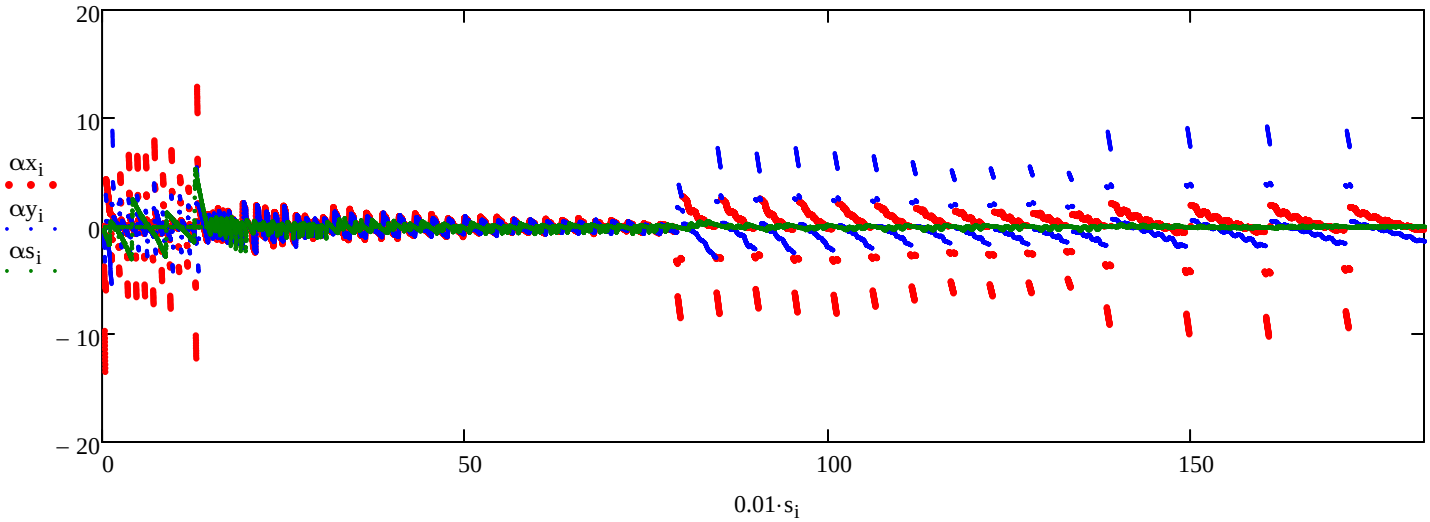


Beta-functions [m]




Beta-function [m]
s [m]
βx
βy
βs
0
10
20
0
50
100
150


Eni / 10^6
βi
0.01·si
1×10^3
900
800
700
600
500
400
300
200
100
0
1
0.8
0.6
0.4
0.2
0
0
50
100
150


αxi
αyi
αsi
0.01·si
20
10
0
− 10
− 20
0
50
100
150

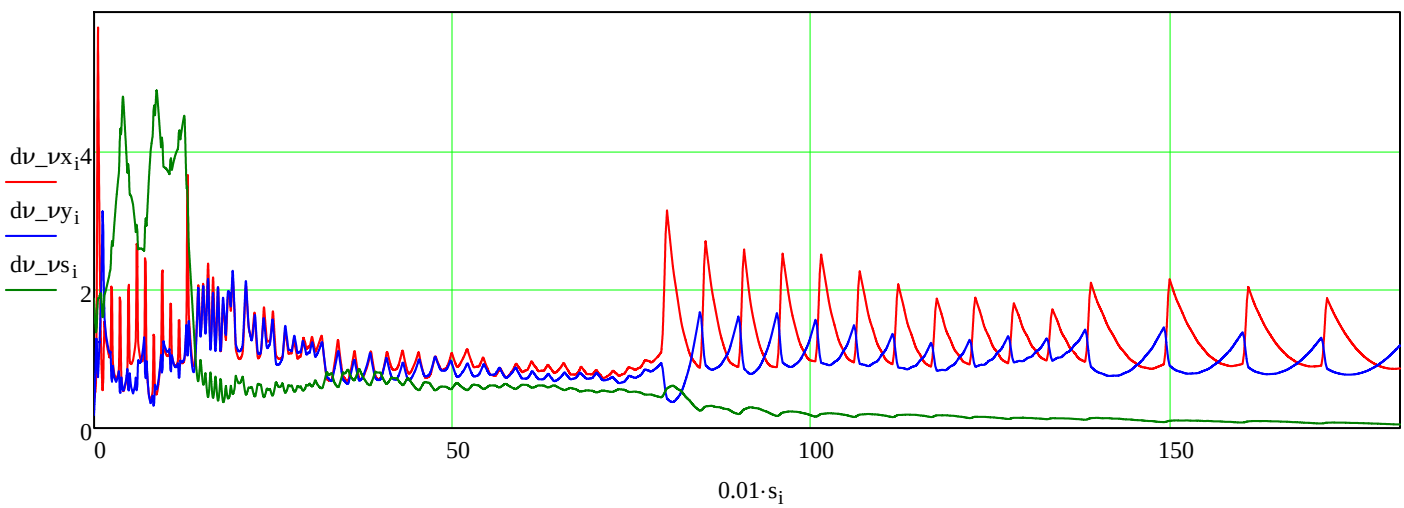
Tune shifts due to beam space charge

$I_b := 5\cdot 10^{-3}\,A$ $\qquad f_b := 162.5\cdot 10^{6}\,Hz$ $\qquad N_p := \dfrac{I_b}{e_{SI}\cdot f_b}$ $\qquad N_p = 1.921\times 10^{8}$

$$dEz_dz(\sigma_x,\sigma_y,\sigma_z) := \frac{4}{\sqrt{\pi}}\cdot\int_0^{\infty}\frac{t^2}{\sqrt{\left(1+2\cdot\sigma_x^{\,2}t^2\right)}\cdot\sqrt{\left(1+2\cdot\sigma_y^{\,2}\cdot t^2\right)}\cdot\left[\sqrt{\left(1+2\cdot\sigma_z^{\,2}\cdot t^2\right)}\right]^3}\,dt$$

Transverse tune shifts: $d\nu_\nu x_i := \dfrac{1}{2}\cdot\dfrac{r_p\cdot N_p\cdot(\beta x_i)^2}{(\beta_i)^2\cdot(\gamma_i)^2}\cdot dEz_dz(\gamma_i\cdot\sigma s_i,\sigma y_i,\sigma x_i)$ $\quad d\nu_\nu y_i := \dfrac{1}{2}\cdot\dfrac{r_p\cdot N_p\cdot(\beta y_i)^2}{(\beta_i)^2\cdot(\gamma_i)^2}\cdot dEz_dz(\gamma_i\cdot\sigma s_i,\sigma y_i,\sigma y_i)$

Longitudinal tune shift: $d\nu_\nu s_i := \dfrac{1}{2}\cdot\dfrac{r_p\cdot N_p\cdot(\beta s_i)^2}{(\beta_i)^2\cdot(\gamma_i)^3}\cdot dEz_dz(\sigma s_i,\sigma y_i,\gamma_i\cdot\sigma s_i)$



Particle loss due to charge stripping intrabeam collisions

Results of numerical simulations from "Stripping H- in low energy collisions with Antiprotons", Phys. Rev. A., v33, N.3, p.33

$\beta_{rel} := \alpha_{FS}\cdot(0.015\ \ 0.02\ \ 0.03\ \ 0.05\ \ 0.1\ \ 0.2\ \ 0.4\ \ 0.6\ \ 1)^T$ $\qquad nh := 0..8$

$\sigma_{sim} := 10^{-16}\cdot(0.4\ \ 20.3\ \ 33.2\ \ 40\ \ 37.7\ \ 34.3\ \ 33.8\ \ 26.5\ \ 19.1)^T$

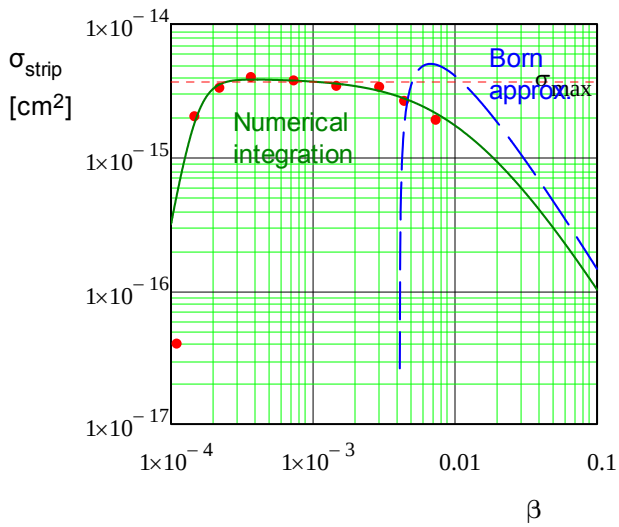
Fitting formulas

$$\sigma_{strip}(\beta, Z) := 152.2\cdot a_0^2\cdot Z^2\cdot\left(\frac{\alpha_{FS}}{\beta}\right)^2\cdot\ln\left[\frac{3.2}{Z}\cdot\left(\frac{\beta}{\alpha_{FS}}\right)^2\right]$$

Theoretical result obtained in born approximation from Нейтрализация ионов H- в столкновениях с быстрыми многозарядными ионами", ЖТФ, 1999, т. 69, в. 7

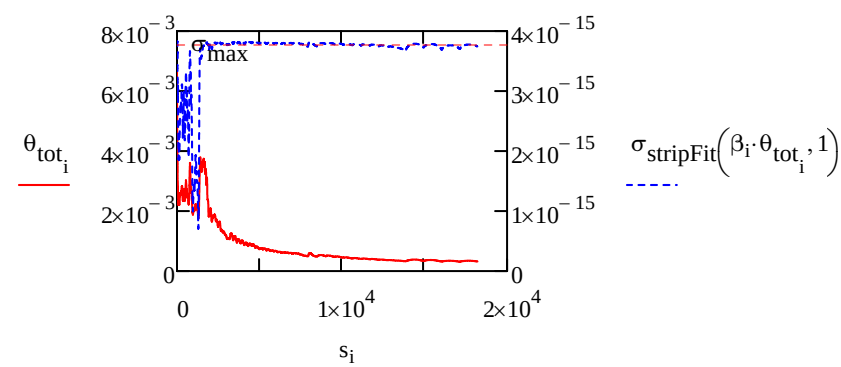
$$\sigma_{stripFit}(\beta, Z) := 120\cdot a_0^2\cdot Z^2\cdot\left(\frac{\alpha_{FS}}{\beta+\alpha_{FS}}\right)^2\cdot\frac{\beta^6}{\left(1.5\cdot10^{-4}\right)^6+\beta^6}\cdot\ln\left[\frac{3.2}{Z}\cdot\left(\frac{\beta+\alpha_{FS}}{\alpha_{FS}}\right)^2\right]$$

$$\sigma_{max} := 135\cdot a_0^2$$



$$\theta_{tot_i} := \sqrt{\left(\sigma_{p_i}\right)^2+\left(\sigma_{\theta x_i}\cdot\gamma_i\right)^2+\left(\sigma_{\theta y_i}\cdot\gamma_i\right)^2}$$

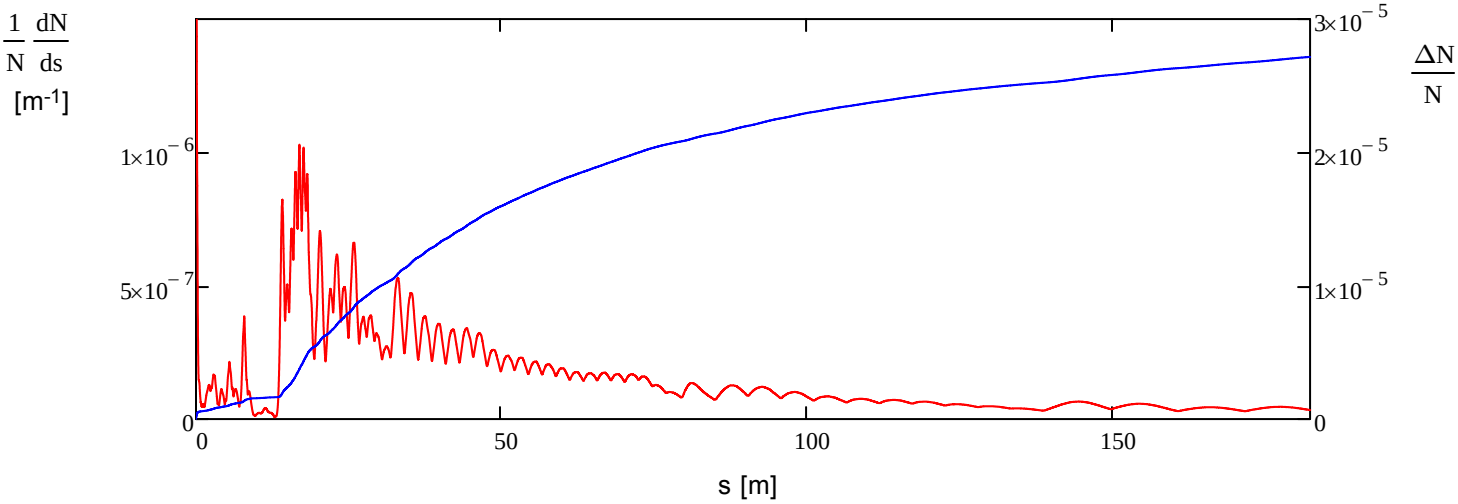
$$\lambda_{L_i} := \frac{N_p\cdot\sigma_{stripFit}\left(\beta_i\cdot\theta_{tot_i},1\right)}{8\cdot\pi^2\cdot\sigma x_i\cdot\sigma y_i\cdot\sigma s_i\cdot\left(\gamma_i\right)^2}\cdot\theta_{tot_i}$$

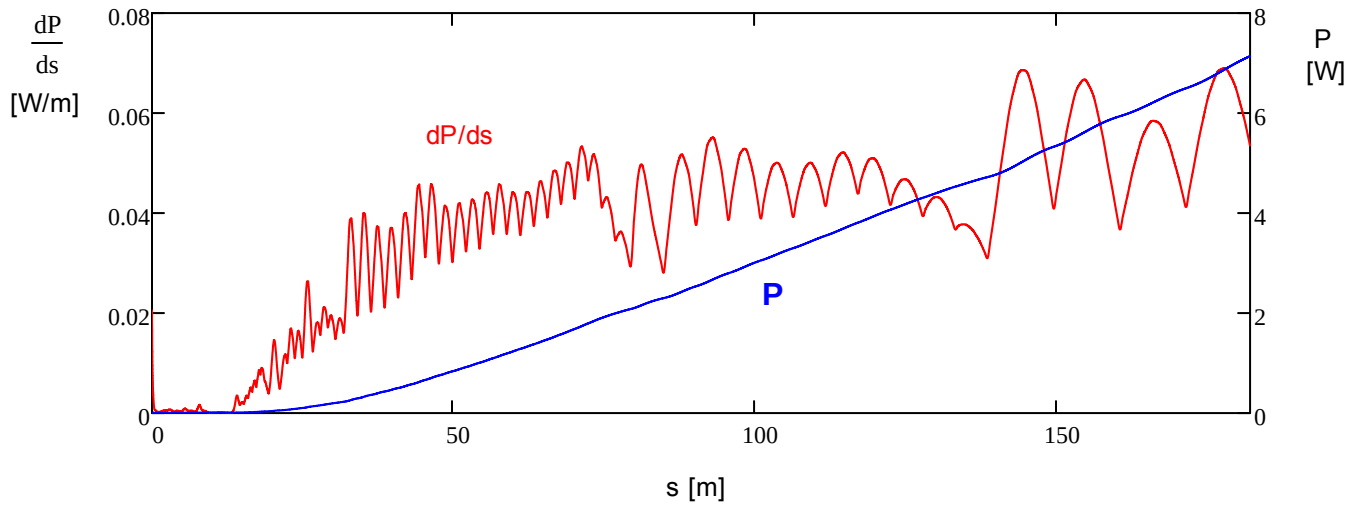


$j := 0..N-2$

$loss_0 := 0 \qquad loss_{j+1} := loss_j + \lambda_{L_j}\cdot\left(s_{j+1}-s_j\right)$

$I_{BeamAv} := 2\cdot10^{-3}A \qquad Ploss_0 := 0 \qquad Ploss_{j+1} := Ploss_j + \lambda_{L_j}\cdot\left(s_{j+1}-s_j\right)\cdot En_j\cdot I_{BeamAv}$





Esimate of radiation in SC linac

Properties of particle passage through iron

$dEdx_0 := 1.45 \cdot 10^6$ MeV/(g/cm²) $\quad L_{nucl} := 132$ g/cm²

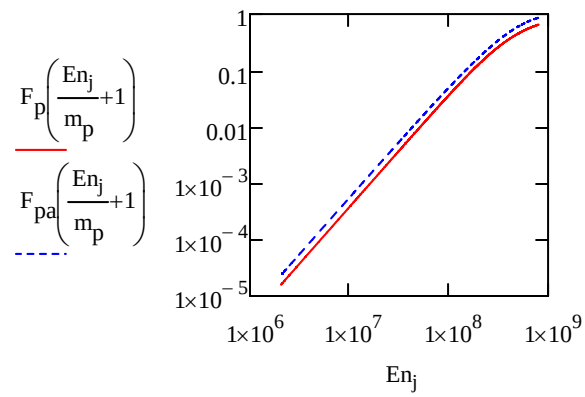
$$F_p(\gamma) := 1 - \frac{1}{\left[1 + \frac{1}{5}\cdot\left(\frac{\gamma - 1}{\gamma_p \cdot L_{nucl}}\right)^2\right]^{0.69}}$$

- I do not remember how this formula was derived

$$\gamma_p := \frac{1}{m_p}\cdot dEdx_0$$

$$F_{pa}(\gamma) := 1 - \exp\left[-\frac{1}{L_{nucl}}\cdot\frac{m_p}{dEdx_0}\cdot\left(\gamma + \frac{1}{\gamma} - 2\right)\right]$$

- this formula is straigtforward to obtain and will be used below



Radiation estimate a formula coming from Tony Leveling

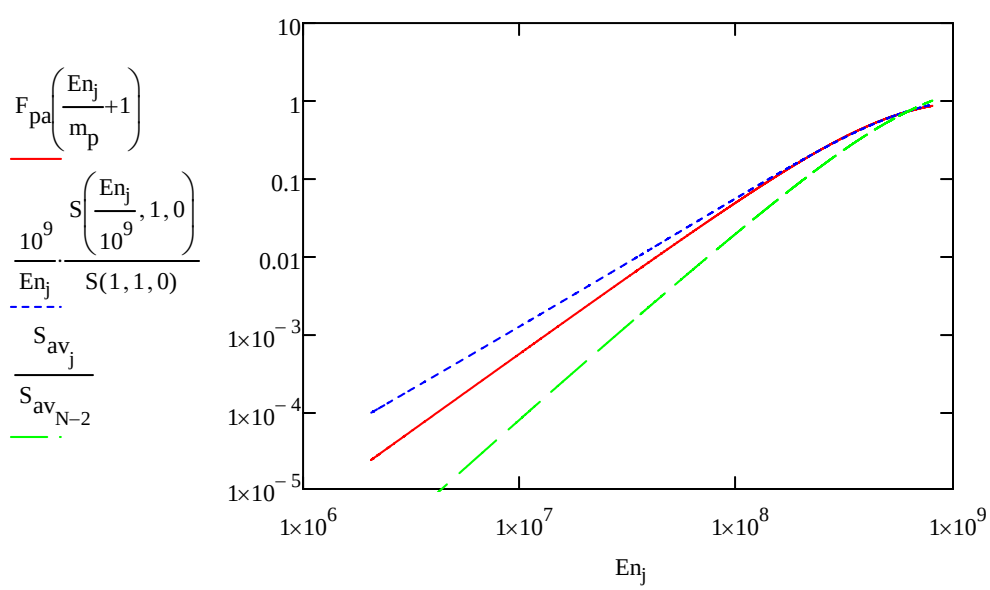
Neutron radiation from primary proton or H- beams dominates shielding concerns for personnel protection.Dose in mrem per proton at a distance r ft from the loss point and at an angle of θs degrees with respect to the beam direction. The proton energy E is measured in units of GeV.

$$S(E, r, \theta_s) := 2\cdot 10^{-5}\cdot\frac{\left(1 + E^{0.6}\right)\cdot\left(1 - \exp\left(-3.6\cdot E^{1.6}\right)\right)}{\left[0.3048\cdot r\cdot\left(\theta_s + \frac{40}{\sqrt{E}}\right)\right]^2}$$

$$S_{av_j} := \frac{1}{2}\cdot\int_0^{\pi}\sin(\theta)\cdot S\left(\frac{En_j}{10^9}, 1, \frac{180}{\pi}\cdot\theta\right)d\theta$$

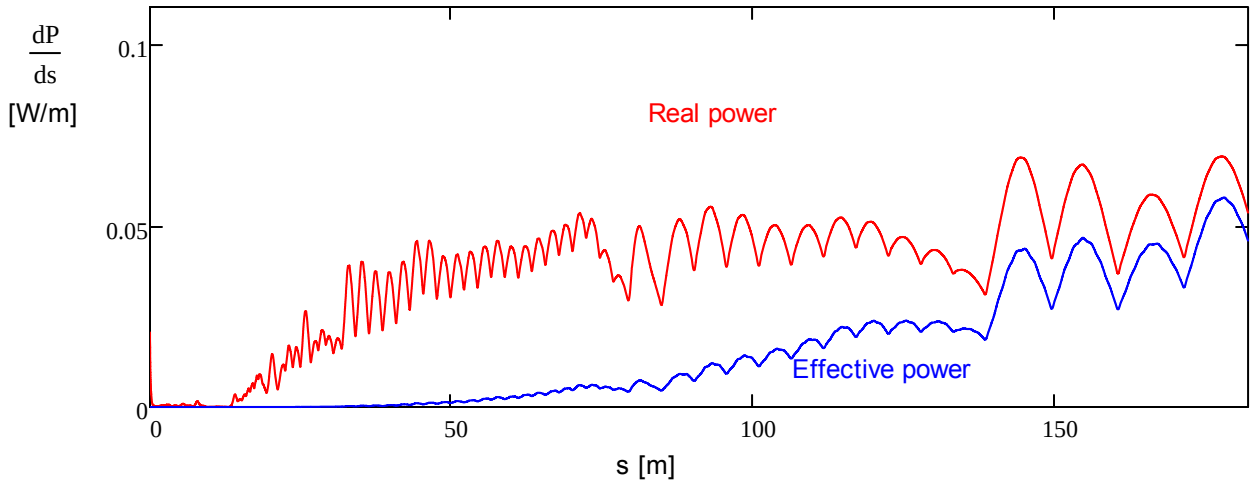
We will use expression for $F_{pa}(\gamma)$ as a conservative estimate

Reduction of radiation with beam energy for fixed beam power

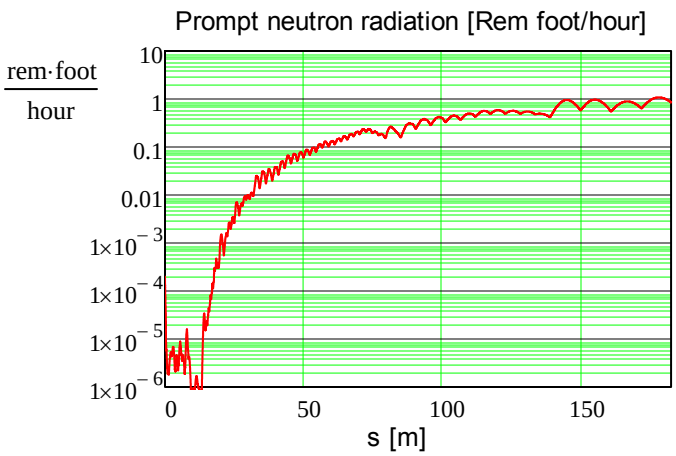


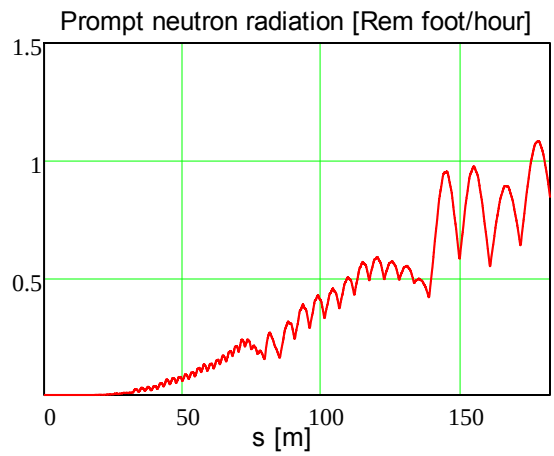
$$PlossEf_0 := 0$$

$$PlossEf_{j+1} := PlossEf_j + F_{pa}\left(\frac{En_j}{m_p} + 1\right)\cdot\lambda_{L_j}\cdot\left(s_{j+1} - s_j\right)\cdot En_j\cdot I_{BeamAv}$$



$$Rad_j := \lambda_{L_j}\cdot\frac{I_{BeamAv}}{e_{SI}}\cdot S_{av_j} \qquad \text{[mrem/s/foot]}$$





worst case estimate we assume that the prompt radiation is 20 rem/hour. Taking into account that 1 rem = 0.01 gray we obtain the worst case estimate for yearly dose: $20\cdot 0.01\cdot 24\cdot 365\cdot 50 = 8.76\times 10^4$ Gy=1.7 kGy*foot/year

$$0.2\cdot 24\cdot 365 = 1.752\times 10^3$$

The following table shows radiation quantities in SI and non-SI units.

Quantity	Name	Symbol	Unit	Year	System
Exposure (X)	röntgen	R	esu / 0.001293 g of air	1928	non-SI
Absorbed dose (D)			$erg \cdot g^{-1}$	1950	non-SI
	rad	rad	$100\ erg \cdot g^{-1}$	1953	non-SI
	gray	Gy	$J \cdot kg^{-1}$	1974	SI
Activity (A)	curie	Ci	$3.7 \times 10^{10}\ s^{-1}$	1953	non-SI
	becquerel	Bq	s^{-1}	1974	SI
Dose equivalent (H)	röntgen equivalent man	rem	$100\ erg \cdot g^{-1}$	1971	non-SI
	sievert	Sv	$J \cdot kg^{-1}$	1977	SI
Fluence (Φ)	(reciprocal area)		cm^{-2} or m^{-2}	1962	SI (m^{-2})