Voltage and Phase Stabilization in SC Cavities

Valeri Lebedev

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Basic Equations

- Actual cavity voltage and current: \( U_c(t) = \text{Re} \left( U(t) e^{i\alpha} \right) \quad I_G(t) = \text{Re} \left( I_g(t) e^{i\alpha} \right) \)

- Change of cavity voltage amplitude is described by:

\[
\frac{dU}{dt} + \frac{\omega_0}{2Q_L} U + i(\Delta \omega + \delta \omega(t))U = \frac{\omega_0 R_L}{2Q_L} (I_g(t) - I_b(t))
\]

where \( U \equiv U(t) \) is the complex cavity voltage amplitude
where \( I_b \equiv I_b(t) \) is the complex current of bunched beam

\[
I_g \equiv I_g(t) = 2 \left( \frac{P \beta}{R_L (1 + \beta)} \right) \quad \text{For SC cavities} \quad \beta \gg 1 \rightarrow 2 \left( \frac{P}{R_L} \right) \quad \text{complex RF generator current}
\]

\[
\phi_{\text{acc}} = \arg \left( \frac{U}{I_b} \right) = \arg(U) - \arg(I_b) \quad \text{- accelerating phase}
\]

\[
Q_L = Q_0 / (1 + \beta) \quad \text{- loaded Q}
\]

\[
\beta \quad \text{- cavity coupling} \quad (\beta = Q_0 / Q_e, \quad 1 / Q_L = 1 / Q_0 + 1 / Q_e)
\]

\[
\omega_0 \quad \text{- desired (reference) frequency}
\]

\[
\Delta \omega \quad \text{and} \quad \delta \omega(t) \quad \text{are the constant cavity frequency offset and the cavity frequency variation due to microphonics and LFD}
\]

That corresponds that

\[
\omega_r = \omega_0 - \Delta \omega - \delta \omega(t) \quad \text{is the cavity resonance frequency}
\]
\[ \frac{dU}{dt} + \frac{\omega_0}{2Q_L} U + i(\Delta\omega + \delta\omega(t)) U = \frac{\omega_0 R_L}{2Q_L} (I_g(t) - I_b(t)) \]

\[ \uparrow \quad \frac{\omega_0}{2Q_L} U + i\Delta\omega U = \frac{\omega_0 R_L}{2Q_L} (I_g - I_b) \]

\[ U = \frac{R_L (I_g - I_b)}{1 + 2iQ_L \frac{\Delta\omega}{\omega_0}} \]

**Static Solution**

For a given voltage amplitude cavity detuning minimizes RF power

\[ P = \frac{R_L (1 + \beta)}{4\beta} |I_g|^2 = \frac{R_L (1 + \beta)}{4\beta} \left( \left( \frac{U}{R_L} + I_b \cos \phi_{acc} \right)^2 + \left( 2Q_L \frac{U \Delta\omega + \delta\omega}{\omega_0} + I_b \sin \phi_{acc} \right)^2 \right) \]

\[ \uparrow \quad \text{Requiring 2-nd addend to be zero for } \delta\omega = 0 \text{ one obtains the optimal cavity detuning:} \]

\[ \Delta\omega = \frac{I_b (R/Q) \sin \phi_{acc}}{\omega_0} = \frac{2U_0}{2U_0} \]

\[ \uparrow \quad \text{Taking into account that } Q_L = Q_0 / (1 + \beta) \text{ and} \]

\[ \text{differentiating over } \beta \text{ one obtains the optimal coupling:} \]

\[ \beta_{opt} = \sqrt{\left( 1 + \frac{I_b \cos \phi_{acc} (R/Q) Q_0}{U_0} \right)^2 + \left( \frac{2\delta\omega Q_0}{\omega_0} \right)^2} \]

**HB650 parameters**

\[ U = 19.9 \text{ MV} \quad f_0 = 650 \text{ MHz} \]

\[ Q_0 = 3 \cdot 10^{10} \quad (R/Q) = 610 \ \Omega \]

Cavity is overcoupled: \( I_{b\_cpl} = 2.6 \text{ mA} \)

\[ \phi_{acc} = 0, \ \delta f = \pm 20 \text{ Hz} \]

\[ \Rightarrow \beta = 3022, \quad R_L = 6.05 \text{ G\Omega}, \]

\[ Q_L = 9.9 \cdot 10^6, \ \Delta f_c = f_0 / Q_L = 65.5 \text{ Hz} \]

\[ I_b = 2 \text{ mA}, \ \phi_{acc} = 15 \text{ deg.} \]

\[ \Rightarrow \Delta f = 5.17 \text{ Hz}, \ P(\delta f = 20Hz) = 47.3\text{ kW} \]
Feedback System with Proportional Gain

- Equation describing parameter deviations relative to static solution
  \[
  \frac{d(U + \delta U)}{dt} + \frac{\alpha_0}{2Q_L}(U + \delta U) + i(\Delta \omega + \delta \omega(t))(U + \delta U) = \frac{\alpha_0 R_L}{2Q_L}(I_g + \delta I_g(t) - I_b - \delta I_b(t))
  \]

- Introduce feedback correction as
  \[
  \delta I_g(t) = -\frac{G}{R_L} \delta U(t - \tau)
  \]

  where \( \tau \) is a delay in the feedback (signal propagation time)

  Such form implies that gains of power and phase corrections are equal
  (perturbation vector in phase space \([|\delta U/U|, \text{arg}(U)]\))

- Accounting for feedback yields
  \[
  \frac{d\delta U}{dt} + \frac{\alpha_0}{2Q_L}(\delta U + G\delta U(t - \tau)) + i(\Delta \omega + \delta \omega(t))\delta U = \frac{\alpha_0 R_L}{2Q_L}(\delta I_g - \delta I_b) - i\delta \omega(t)U
  \]
**System Stability with Proportional Feedback**

- In the absence of external perturbations in Fourier harmonics we have

\[
\dot{U}_\omega + \left( \frac{\omega_0}{2Q_L} \left( 1 + Ge^{-i\omega \tau} \right) + i \Delta \omega \right) \delta U_\omega = 0 \quad \text{for } \frac{G}{Q_L} \gg 1 \Rightarrow \omega_0 \tau = 0 \Rightarrow \sin \omega \tau = 1 \Rightarrow i\omega + \frac{\omega_0 G}{2Q_L} (\cos \omega \tau + i \sin \omega \tau) = 0
\]

- Stability will be lost when with increase of \( G \) the above equation gets a real solution for \( \omega \)

- If \( \tau \ll Q_L / \omega_0 \) then \( G \) is large and we can neglect \( \Delta \omega \)

- The maximum gain is \( G_{\text{max}} = \pi Q_L / (\omega_0 \tau) \)

  Corresponding instability is developed at frequency \( \omega_{\text{inst}} = \pi / (2\tau) \)

- In a practical system we need to be sufficiently far from instability threshold

  - In below estimates we use \( G = 0.4G_{\text{max}} \approx 1.26Q_L / (\omega_0 \tau) \)

- For HB650 cavity it yields \( G = 3054 \)

  I.e. feedback system increases the natural damping by about 3000 times
**Suppression of Beam Current Oscillations with Proportional Feedback**

- Beam current fluctuations excite cavity voltage

\[
\delta U_\omega = \frac{-R_I \delta I_{b\omega}}{1 + Ge^{-i\omega \tau} + 2iQ_L \frac{\Delta \omega + \omega}{\omega_0}}
\]

**Suppression of Fourier harmonic of cavity voltage excited by beam current with different feedback gains for HB650 cavity. The same data are presented at both plots.**

- Beam current fluctuations produce too large voltage fluctuations if not suppressed by feedback system
**Feedback System of Integrating Type**

- Introduce feedback correction as

\[
\delta I_g(t) = -\frac{G}{R_L} \left( \delta U(t - \tau) + \frac{1}{\tau_I} \int_{-\infty}^{t-\tau} \delta U(t') dt' \right)
\]

where \( \tau \) is a delay in the feedback (signal propagation time),
\( G \) is the gain of proportional system,
\( \tau_I \) is the integration time constant.

Actually the system is a combination of integrating and proportional feedbacks.
Same as before such form implies that gains of power and phase corrections are equal. We can rewrite the above equation for feedback signal in Fourier harmonics:

\[
i\omega \delta I_{g\omega} = -\frac{G}{R_L} \left( 1 + \frac{1}{i\omega \tau_I} \right) e^{-i\omega \tau} \delta U_\omega
\]

\( \Rightarrow \)

For beam current perturbation

\[
\delta U_\omega = \frac{-R_L \delta I_{b\omega}}{1 + G \left( 1 + \frac{1}{i\omega \tau_I} \right) e^{-i\omega \tau} + 2iQ_L \frac{\Delta \omega + \omega}{\omega_0}}
\]

For cavity frequency perturbation

\[
\delta U_\omega = \frac{-2iQ_L R_L \delta f_\omega}{f_0 \left( 1 + G \left( 1 + \frac{1}{i\omega \tau_I} \right) e^{-i\omega \tau} + 2iQ_L \frac{\Delta \omega + \omega}{\omega_0} \right)}
\]
**Effective Electronic Gain of Integrating Type Feedback**

\[
G_{\text{tot}}(\omega) = G \left( 1 + \frac{1}{i \tau \omega} \right) e^{-i \omega \tau}
\]

*Dependence of effective feedback gain on frequency for HB650 cavity; \( \tau = 20 \mu s \)*

- The integration time constant is chosen to maximize the gain at low frequencies but so that at high frequencies only proportional gain would be left. The proportional gain determines beam stability.
Gains presented in Brian’s Talk at

Progress to date: Open loop transfer function simulation of cavity and controller

Max gain
Closed-loop bandwidth: ~50 kHz
Control system zero: 15 kHz
Proportional gain: 1500
Integral gain: 1.44e+08

Nominal gain
Closed-loop bandwidth: ~25 kHz
Proportional gain: 750
Integral gain: 7e+07
Suppression of Beam Current Oscillations with Integrating Type Feedback

Suppression of Fourier harmonic of cavity voltage excited by beam current with different feedback gains for HB650 cavity, $I_\omega/I=0.01$, $\tau_I=20$ $\mu$s. The same data are presented at both plots.

- 1% current oscillation excites $\delta U/U \approx 3 \cdot 10^{-6}$ at maximum
  - Corresponding oscillation of the phase is $1.7 \cdot 10^{-4}$ deg.
- It is well within specs
Suppression of Cavity Frequency Oscillations with Integrating Type Feedback

Suppression of Fourier harmonic of cavity voltage excited by cavity frequency oscillations with different feedback gains for HB650 cavity, $\delta \omega = 20$ Hz, $\tau_I = 20$ $\mu$s

- Cavity frequency oscillation with 20 Hz amplitude excites $\delta U/U \approx 2 \cdot 10^{-5}$ in the frequency band of 1 kHz
  - Corresponding oscillation in the phase is $1.1 \cdot 10^{-3}$ deg.
- It is expected to be the main source of voltage ripple
  - However, it is well within specs

Beam Current Measurements with Resistive Wall Monitor, Valeri Lebedev, Fermilab, September 12, 2017
**Noise of Feedback System**

- Spectral density of relative cavity voltage

\[ S_U(f) = \left( \frac{\delta U}{U} \right)_f^2 = \frac{G^2 S_{\text{noise}}(f)}{1 + G \left( 1 + \frac{1}{2 \pi f \tau_1} \right) e^{-2 \pi f \tau} + 2 i Q_L \Delta f + f} \]

where \( S_{\text{noise}}(f) \) is the spectral density of relative measurement noise

For FNAL LLRF \( S_{\text{noise}}(f) = 155 \text{ dBC/Hz} \)

- Corresponding rms relative voltage fluctuations are

\[ \left( \frac{\delta U}{U} \right) = \int_0^\infty S_U(f) df \Rightarrow \sqrt{\left( \frac{\delta U}{U} \right)^2} = 10^{-5} \]

- Thus, the voltage fluctuations due to measurement noise create voltage fluctuations well within specs
**Collective Response of PIP-II Linac**

- Propagation of perturbation excited by momentum perturbation of $\frac{\delta p}{p_0} = 0.1\%$ before the first HWR (top) and SSR1 (bottom) cavities
  - Propagation of perturbation is amplified in the absence of feedback
  - Amplification excited by a cavity decreases fast with cavity number (energy)
  - Feedback with $G>10$ (for each Cavity) suppresses this amplification (collective response of the linac)
**Measurement of Cavity Detuning**

\[
\frac{dU}{dt} + \frac{\omega_0}{2Q_L}U + i(\Delta \omega + \delta \omega(t))U = \frac{\omega_0 R_L}{2Q_L}(I_g(t) - I_b(t))
\]

- LLRF stabilizes RF voltage and its phase \(\Rightarrow dU/dt=0\) (also sets \(\text{Im}(U)=0\))
- In absence of microphonics and beam current fluctuations we detune cavity by \(\Delta \omega = \omega_0 R_L I_{b0} \sin \varphi_{acc} / (2Q_L U)\) (that is the static detune)

\[
\Rightarrow |I_{g0}| = \frac{U}{R_L} + I_{b0} \cos \varphi_{acc}, \quad \varphi_{g0} = 0
\]

- Microphonics and beam current fluctuations result in

\[
\varphi_g \approx \frac{1}{U + R_L I_{b0} \cos \varphi_{acc}} \left( \frac{2Q_L \delta \omega(t)}{\omega_0} - R_L \delta I_b(t) \sin \varphi_{acc} \right)
\]

- Inverting

\[
\delta \omega(t) = \frac{\omega_0 R_L}{2Q_L U} \left( \sin \varphi_{acc} \delta I_b(t) + I_{b0} \cos \varphi_{acc} \delta \varphi_{acc}(t) + \left( I_{b0} \cos \varphi_{acc} + \frac{U}{R_L} \right) \delta \varphi_g(t) \right)
\]

- For small deviations the changes of beam current and errors in setting accelerating and generator phases result in an error in cavity frequency computation
Measurement of Cavity Detuning (2)

- For HB resonators and $\varphi_{acc} = 15$ deg:
  - $\delta I_b/I_b = 0.05 \Rightarrow \delta \omega = 0.26$ Hz
  - $\delta \varphi_{acc} = 2$ deg. $\Rightarrow \delta \omega = 0.67$ Hz
  - $\delta \varphi_g = 2$ deg. $\Rightarrow \delta \omega = 1.8$ Hz

- To set reference for $\varphi_g$
  - Switch off the beam and reduce voltage so that microphonics would not trip the beam. Microphonics suppression is off.
  - Adjust reference so that the minimum power would be at $\varphi_g = 0$
  - Expected accuracy is ~ 1 deg. for relative power measurement of $10^{-4}$

- To set reference for $\varphi_{acc}$
  - Rotate cavity phase by 360 deg. and observe the velocity change using downstream BPMs
  - Expected accuracy depends on cavity location (~1 deg may be achieved)
    - Large microphonics will lead to small accelerating voltage
Alternative Proposal for Cavity Detuning Measurement

- Introduce a phase modulation of RF power with amplitude $\theta_g$ and with sufficiently high frequency (in the range of 20-100 kHz to be well above the microphonics frequencies and cavity bandwidth)
  - For $\theta_g < 0.1$ the corresponding power increase $P = P_0(1 + \theta_g^2/2)$ is negligible

- Phase modulation of RF power will result in a modulation of RF voltage proportional to the detuning, $\Delta f_0$
  - For sufficiently high perturbation frequency, $f_m \gg f_0/Q_L, \Delta f_0$, one obtains
    - RF voltage phase modulation in the cavity
    - and corresponding amplitude of cavity voltage modulation

$$\theta_U = \text{Im} \left( \frac{\delta U}{U} \right) = \theta_g \frac{f_0}{Q_L f_m}$$

$$\text{Re} \left( \frac{\delta U}{U} \right) = -\theta_g \frac{2 \Delta f_0}{f_m}$$

As one can see the voltage modulation is proportional to cavity detuning and changes its sign together with sign of detuning

This signal can be used in the resonance control feedback

- Note that both amplitude and phase perturbations are shifted from RF generator phase perturbation by 90 deg.
Alternative Proposal for Cavity Detuning Measurement

- The phase perturbation has to be small enough so that oscillation of voltage and phase would be within specs: 0.01%, 0.01 deg
  - For $\theta_g=4.6$ deg,
    - $f_m=30$ kHz and
    - nominal HB cavity detuning $\Delta f_0=5.1$ Hz
  One obtains:
  - $\Delta U/U=2.7\cdot 10^{-5}$
  - $\theta_U=0.01$ deg

- The rms noise in 2 kHz band is $\Delta U/U =0.84\cdot 10^{-6}$
  - It results in an accuracy of the cavity detuning measurement <0.2 Hz

- Note that the modulation frequency should not coincide with frequencies presented in the beam which can strongly affect or bias the measurements results

Time dependence of relative cavity voltage: red - voltage phase, blue and green - voltage amplitudes for nominal HB cavity detuning and its negative value; Modulation frequency - 30 kHz; Amplitude of phase modulation $\theta_g=4.6$ deg;
Conclusions

- In any scenario each LLRF station has to know the beam current
  - It is better if one device is used for all cavities
- If we use the phase difference between the RF voltage and the drive we need to think how the reference phases for RF power and accelerating voltage can be set and supported for long time in the presence of large microphonics
- Alternative measurement of cavity detuning does not have these problems
  - It does not depend on the cavity and beam parameters and is expected to have better accuracy
    \[
    \Delta f_0 = -\frac{f_m}{2\theta_g} \text{Re}\left(\frac{\delta U}{U}\right)
    \]
  - Very important advantage is that if we know how much a cavity is detuned we can compute and control the accelerating phase
- Presently, we plan to have a possibility of cavity phase and amplitude perturbations
  - It can be used for multiple purposes
Backup slides
Beam Current Measurements with Resistive Wall Monitor, Valeri Lebedev, Fermilab, September 12, 2017

Beam Current Fluctuations Measured in PIP2IT

- One can clearly see current fluctuations of ~1-2% in the course of each pulse as well as pulse to pulse variations
Comments on Cavity Voltage Stabilization

- The LLRF proposed for cavity voltage stabilization keeps voltage fluctuations well within specifications (0.01%, 0.01 deg.).
- The problem we need to address is how to keep each cavity tuned within required band ±20 Hz so that to minimize required power of RF power amplifiers.
- There are only 2 parameters which can be measured in the cavity: they are (1) the RF voltage amplitude and (2) the RF voltage phase.
  - The phase and amplitude of reflected signal are directly determined by the power and phase of RF generator and the voltage and phase of cavity probe signal (Reflected power does not directly depend on beam current).
- In the general case there are 3 parameters which need to be corrected: they are (1) the RF voltage amplitude, (2) the RF voltage phase and (3) the cavity resonant frequency.
  - First two parameters are controlled directly by LLRF.
  - The cavity resonant frequency can be controlled directly from (1) measurements of phase difference between the generator current and RF voltage, and (2) by introducing small phase modulation of generator power.
Suppression of Cavity Frequency Oscillations with Proportional Feedback

\[
\delta U_\omega = \frac{-2iQ_L R_L \delta f_\omega}{f_0 \left(1 + Ge^{-i\omega \tau} + 2iQ_L \frac{\Delta \omega + \omega}{\omega_0}\right)}
\]

Suppression of Fourier harmonic of cavity frequency variation (\(\delta f = 20 \text{ Hz}\)) with different feedback gains for HB650 cavity. The same data are presented at both plots.

- Cavity frequency fluctuations produce quite large voltage fluctuations even if suppressed by feedback system.
Positive and Negative Frequencies

- The initial equation for voltage perturbation is complex. Therefore, the solutions for negative and positive frequencies ($\delta U(t)e^{i\omega t}, \delta U(t)e^{-i\omega t}$) are not complex conjugate and one needs to be accurate in an interpretation of the results.

- A solution for positive frequency actually implies that the current perturbation is rotating in the complex plane with frequency $\omega$; and a solution with negative frequencies implies that the current perturbation is rotated in the opposite direction.

- If the perturbation is “real” ($\cos(\omega t)$ or $\sin(\omega t)$) one needs to combine positive and negative frequencies.

  - Assume that a single frequency solution is: $\delta U(t) = K(\omega)e^{i\omega t} \delta I_{g0}$
  - Then for cos like perturbation one can write
    $$\delta I_g(t) = \delta I_{g0} \cos(\omega t) = I_{g0} \frac{e^{i\omega t} + e^{-i\omega t}}{2} \Rightarrow \delta U(t) = I_{g0} \frac{K(\omega)e^{i\omega t} + K(-\omega)e^{-i\omega t}}{2}$$
  - This solution still implies that both perturbations in the current and voltage may be complex numbers.

- If not directly mentioned, below we consider a response to $e^{i\omega t}$ perturbation.
Measurement of Cavity Detuning

\[ \frac{dU}{dt} + \frac{\alpha_0}{2Q_L} U + i(\Delta \omega + \delta \omega(t)) U = \frac{\alpha_0 R_L}{2Q_L} (I_g(t) - I_b(t)) \]

- LLRF stabilizes RF voltage and its phase => \( \frac{dU}{dt} = 0 \) (also sets \( \text{Im}(U) = 0 \))

\[ |I_g(t)| = \sqrt{\left( \frac{U}{R_L} + I_b(t) \cos \varphi_{acc} \right)^2 + \left( \frac{2Q_L (\Delta \omega + \delta \omega(t))}{\omega_0 R_L} - I_b(t) \sin \varphi_{acc} \right)^2} \]

\[ \varphi_g = \arctan \left( \frac{\frac{2Q_L (\Delta \omega + \delta \omega(t))}{\omega_0} - R_L I_b(t) \sin \varphi_{acc}}{U + R_L I_b(t) \cos \varphi_{acc}} \right) \]

- In absence of microphonics and beam current fluctuations we detune cavity by \( \Delta \omega = \omega_0 R_L I_{b0} \sin \varphi_{acc} / (2Q_L U) \) (that is the static detune)

\[ |I_{g0}| = \frac{U}{R_L} + I_{b0} \cos \varphi_{acc} , \quad \varphi_{g0} = 0 \]

- Microphonics and beam current fluctuations result in

\[ |I_g(t)| = \sqrt{\left( \frac{U}{R_L} + (I_{b0} + \delta I_b(t)) \cos \varphi_{acc} \right)^2 + \left( \frac{2Q_L \delta \omega(t)}{\omega_0 R_L} - \delta I_b(t) \sin \varphi_{acc} \right)^2} \]

\[ \varphi_g \approx \frac{1}{U + R_L I_{b0} \cos \varphi_{acc}} \left( \frac{2Q_L \delta \omega(t)}{\omega_0} - R_L \delta I_b(t) \sin \varphi_{acc} \right) \]

- For \( \delta I(t) = 0 \) the minimum of RF power is achieved for \( \delta \omega(t) = 0 \)
Measurement of Cavity Detuning (2)

- For small deviations the changes of beam current and errors in setting accelerating and generator phases result in an error in cavity frequency computation

\[ \delta \omega(t) = \frac{\omega_0 R_L}{2Q_L U} \left( \delta I_b(t) \sin \varphi_{acc} + I_{b0} \cos \varphi_{acc} \delta \varphi_{acc}(t) + \left( I_{b0} \cos \varphi_{acc} + \frac{U}{R_L} \right) \delta \varphi_g(t) \right) \]

- For HB resonators and \( \varphi_{acc} = 15 \) deg:
  - \( \delta I_b/I_b = 0.05 \) => \( \delta \omega = 0.26 \) Hz
  - \( \delta \varphi_{acc} = 2 \) deg. => \( \delta \omega = 0.67 \) Hz
  - \( \delta \varphi_g = 2 \) deg. => \( \delta \omega = 1.8 \) Hz

- To set reference for \( \varphi_g \)
  - Switch off the beam and reduce voltage so that microphonics would not trip the beam. Microphonics suppression is off.
  - Adjust reference so that the minimum power would be at \( \varphi_g = 0 \)
  - Expected accuracy is ~ 1 deg. for relative power measurement of \( 10^{-4} \)

- To set reference for \( \varphi_{acc} \)
  - Rotate cavity phase by 360 deg. and observe the velocity change using downstream BPMs
  - Expected accuracy depends on cavity location (~1 deg may be achieved)
    - Large microphonics will lead to small accelerating voltage
Measurement of Cavity Detuning (3)

Contribution of different addends to a measurement of cavity detuning angle for all SRF cavities:
- (red) $\delta I_b/I_b = 0.05$
- (blue) $\delta \varphi_{acc} = 2\,\text{deg}$
- (green) $\delta \varphi_g = 2\,\text{deg}$