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A short update on the Beam Energy measurement from PI Test RFQ

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On behalf of PI Test Group
PIP-II technical meeting
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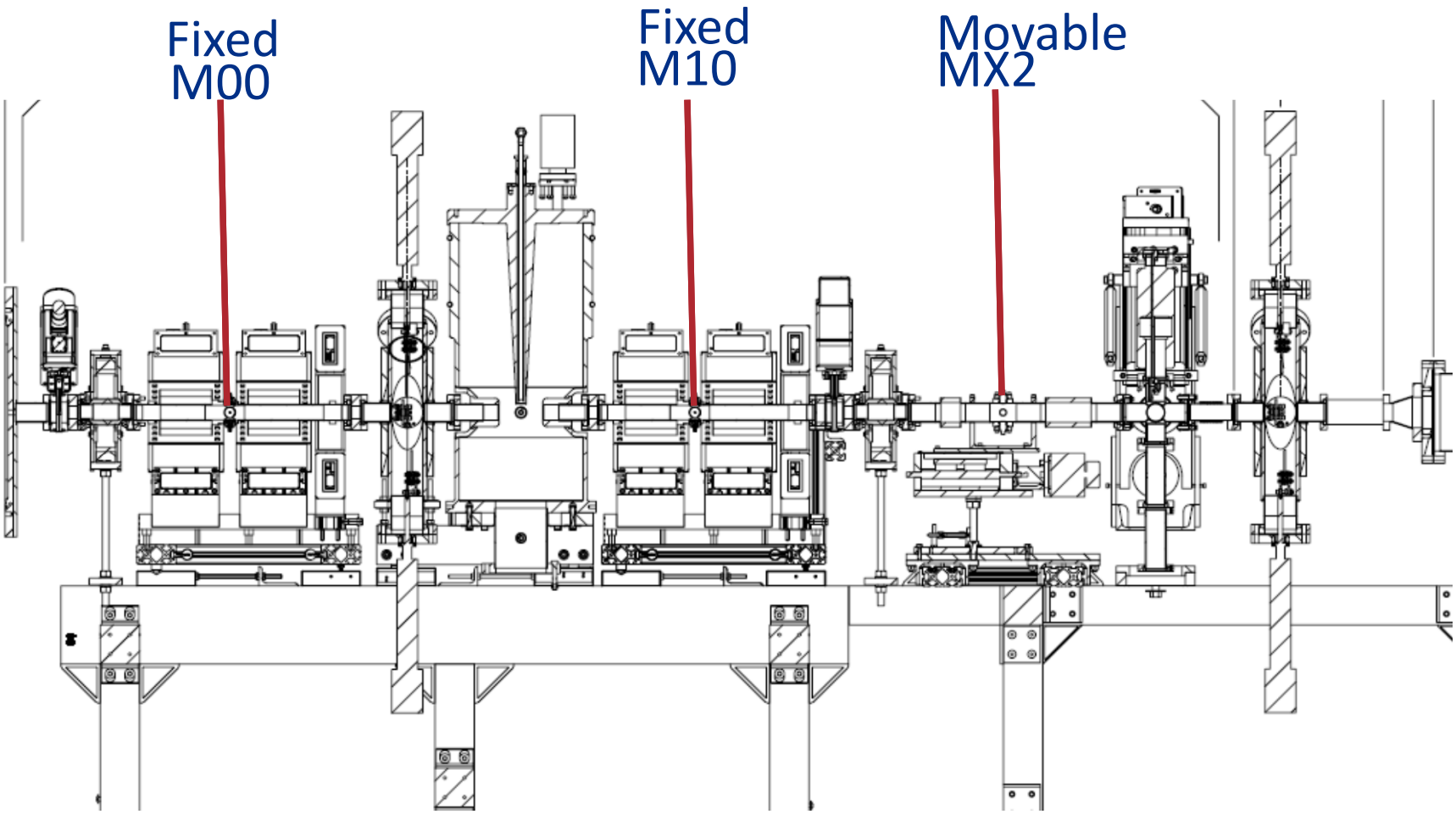
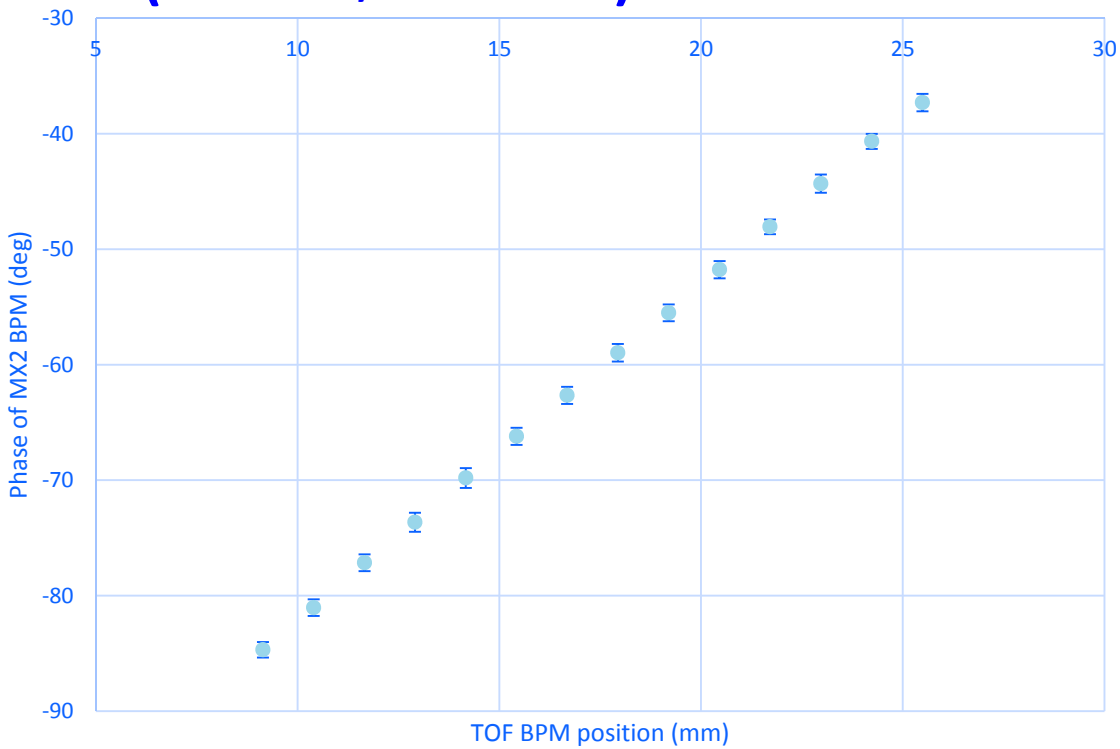
Outline

- **Mistakes done in beam energy error measurements**
  - Using LINEST function in EXCEL
  - Standard Statistical calculations (Bevington)
- **Possible errors in the measurements**
- **Views on using fixed BPMs for energy measurements**
- **Summary**

Layout of the PI-Test MEBT 1.1



Energy measurement data taken on 2nd June, 2016 using Movable BPM (data taken by me & Jonathan)



Least Square fit

$$a = \frac{1}{\Delta}\begin{vmatrix} \sum \frac{y_i}{\sigma_i^2} & \sum \frac{x_i}{\sigma_i^2} \\ \sum \frac{x_i y_i}{\sigma_i^2} & \sum \frac{x_i^2}{\sigma_i^2} \end{vmatrix} = \frac{1}{\Delta}\left(\sum \frac{x_i^2}{\sigma_i^2} \sum \frac{y_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \sum \frac{x_i y_i}{\sigma_i^2}\right)$$

$$b = \frac{1}{\Delta}\begin{vmatrix} \sum \frac{1}{\sigma_i^2} & \sum \frac{y_i}{\sigma_i^2} \\ \sum \frac{x_i}{\sigma_i^2} & \sum \frac{x_i y_i}{\sigma_i^2} \end{vmatrix} = \frac{1}{\Delta}\left(\sum \frac{1}{\sigma_i^2} \sum \frac{x_i y_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \sum \frac{y_i}{\sigma_i^2}\right)$$

$$\Delta = \begin{vmatrix} \sum \frac{1}{\sigma_i^2} & \sum \frac{x_i}{\sigma_i^2} \\ \sum \frac{x_i}{\sigma_i^2} & \sum \frac{x_i^2}{\sigma_i^2} \end{vmatrix} = \sum \frac{1}{\sigma_i^2} \sum \frac{x_i^2}{\sigma_i^2} - \left(\sum \frac{x_i}{\sigma_i^2}\right)^2$$

$$\sigma_b^2 \simeq \sum_{j=1}^{N} \frac{\sigma_j^2}{\Delta^2}\left[\frac{x_j^2}{\sigma_j^4}\left(\sum \frac{1}{\sigma_i^2}\right)^2 - \frac{2x_j}{\sigma_j^4} \sum \frac{1}{\sigma_i^2} \sum \frac{x_i}{\sigma_i^2} + \frac{1}{\sigma_j^4}\left(\sum \frac{x_i}{\sigma_i^2}\right)^2\right]$$

$$= \frac{1}{\Delta^2}\left[\sum \frac{x_j^2}{\sigma_j^2}\left(\sum \frac{1}{\sigma_i^2}\right)^2 - 2\sum \frac{x_j}{\sigma_j^2} \sum \frac{1}{\sigma_i^2} \sum \frac{x_i}{\sigma_i^2} + \sum \frac{1}{\sigma_j^2}\left(\sum \frac{x_i}{\sigma_i^2}\right)^2\right]$$

$$= \frac{1}{\Delta^2}\left(\sum \frac{x_j^2}{\sigma_i^2}\right)\left[\sum \frac{1}{\sigma_j^2} \sum \frac{1}{\sigma_i^2} - \left(\sum \frac{x_i}{\sigma_i^2}\right)^2\right]$$

$$= \frac{1}{\Delta} \sum \frac{1}{\sigma_i^2}$$

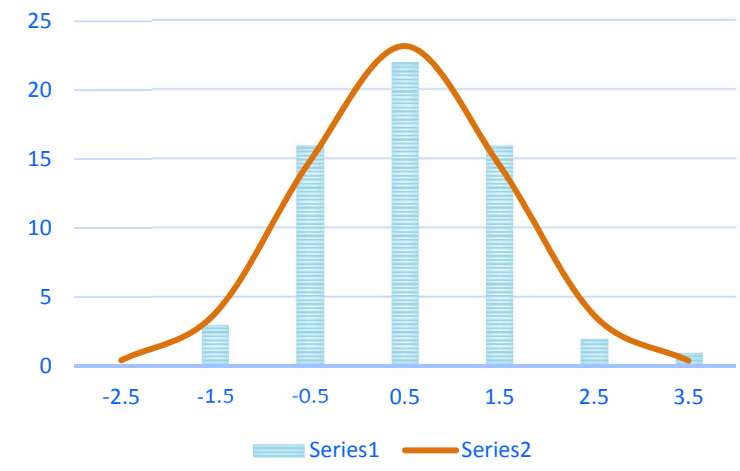
Errors in the slope:

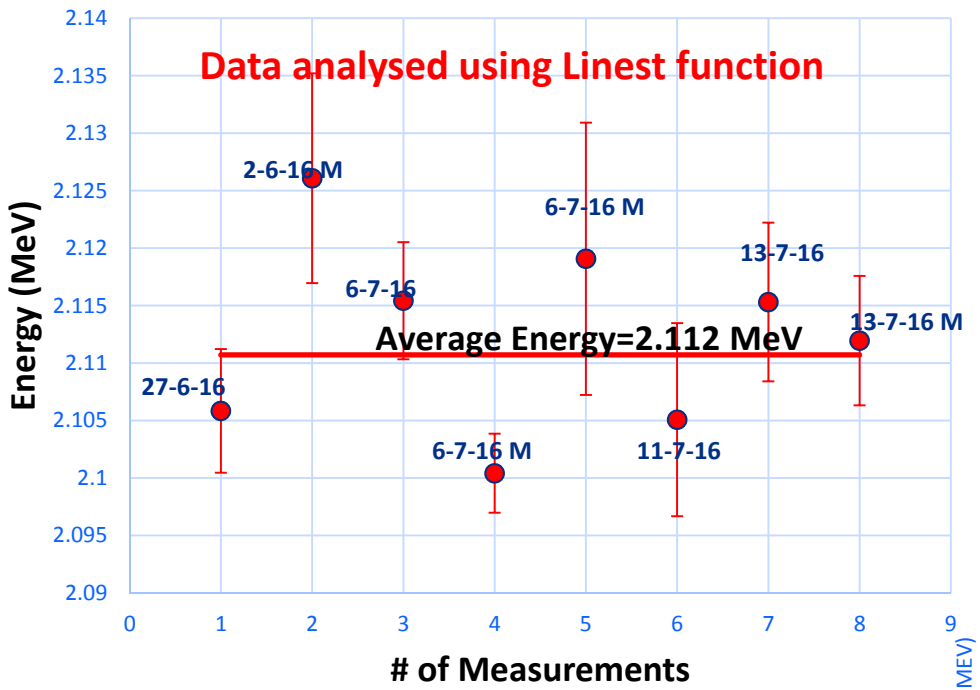
Linest function: 2.9031±0.006 deg/mm → **2.126±0.009 (0.43%) MeV**

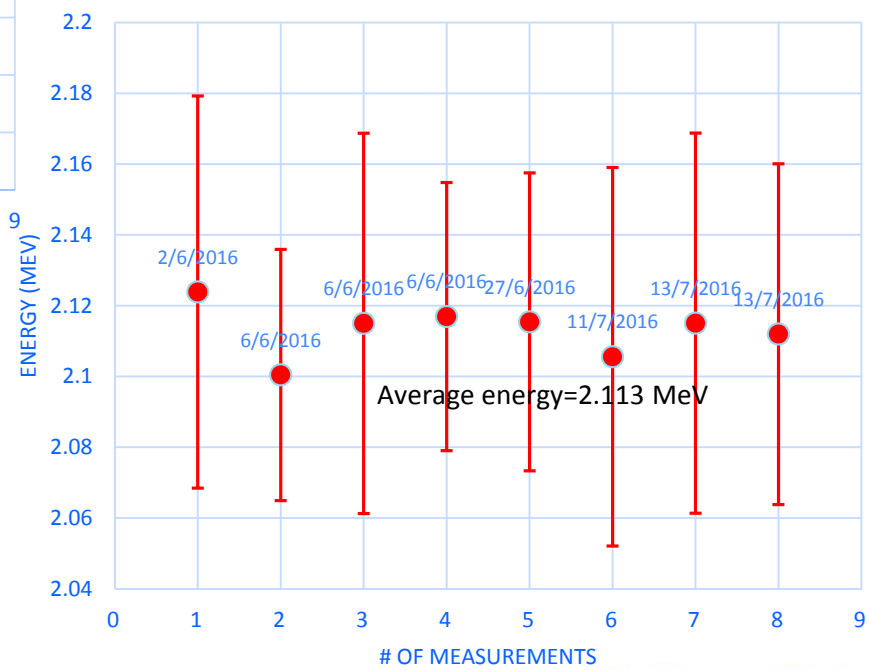
Statistical analysis: 2.9047±0.038 deg/mm → **2.12±0.06 (2.6%) MeV**

$$E(MeV) = \left[\frac{slope}{\sqrt{slope^2 - 0.195^2}} - 1\right] * m_0 c^2$$

We plotted the distribution of phase: Looks Gaussian

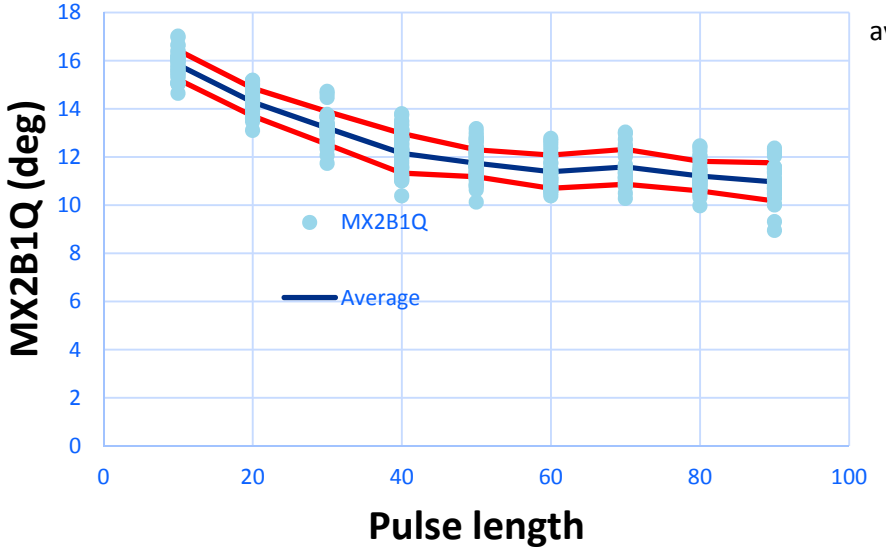






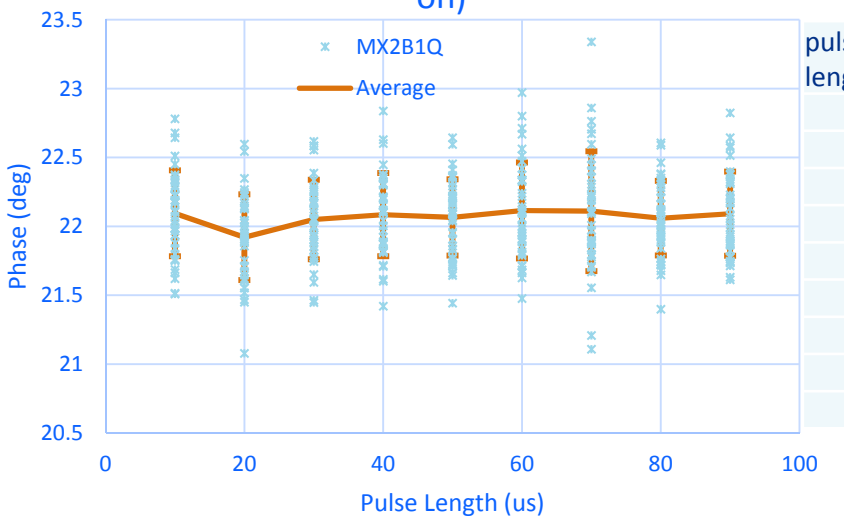
The analysis showed the error bar is above the specs

Then we used the buncher on and at -90 deg and BC on



average	std deviation
15.82719	0.605806492
14.27951	0.57695707
13.20389	0.679128338
12.15142	0.823795615
11.73801	0.562310434
11.38634	0.68901127
11.58904	0.726589599
11.20642	0.616164666
10.96233	0.789549944

TOF BPM phase vs pulse length (buncher on)

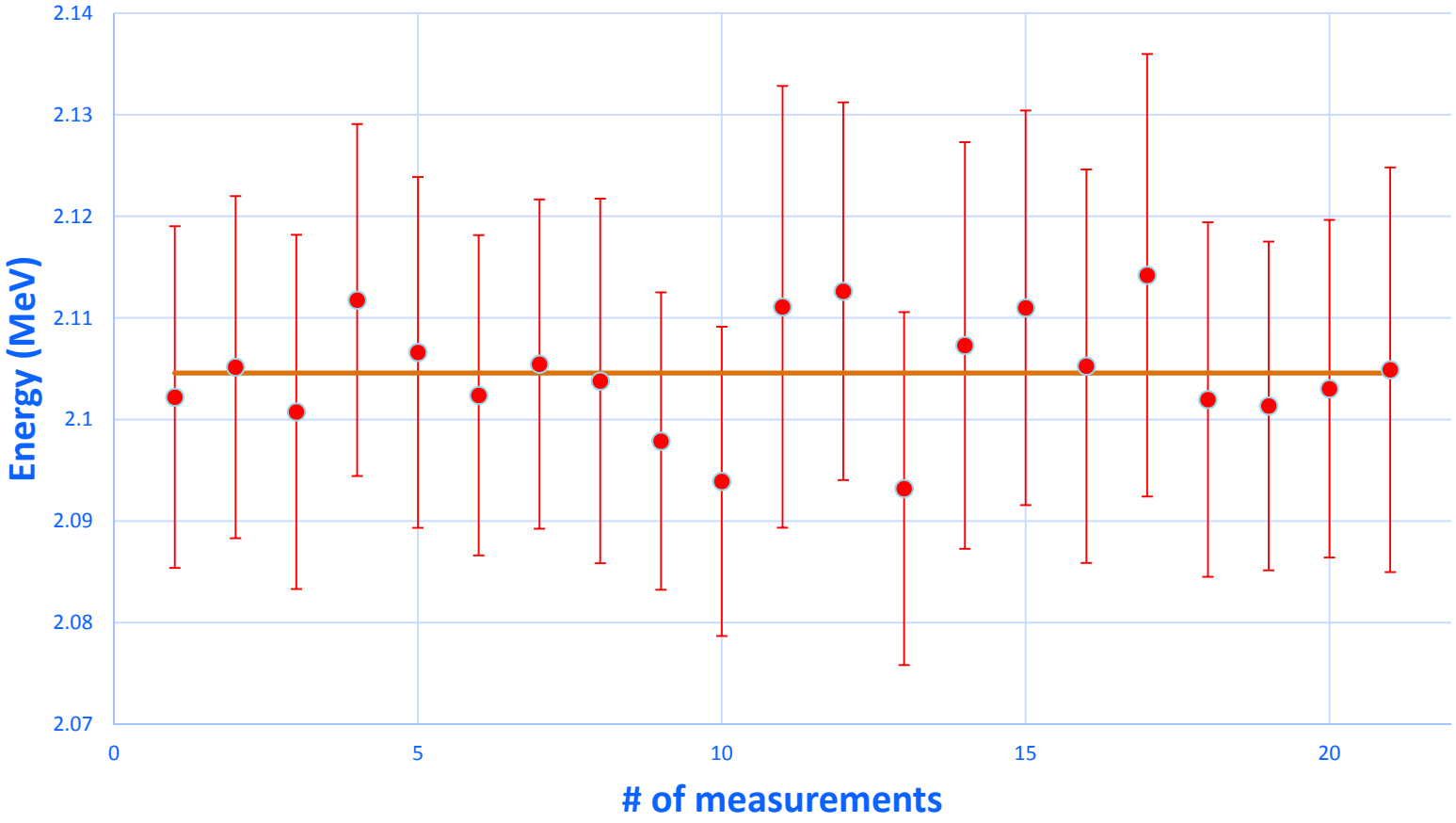


pulse length	average(P:MX2B1Q)	
10	22.09405503	0.312216231
20	21.92044053	0.311204415
30	22.04911532	0.288926226
40	22.08448415	0.302196088
50	22.06443119	0.277223333
60	22.11458774	0.348654308
70	22.10965147	0.434989253
80	22.05892396	0.270157342
90	22.09137177	0.305613146

- ✓ **Standard deviation in phase reduced by switching on the buncher and at -90 deg phase (Bunching phase) and BC on this reduced the energy error.**
- ✓ **We have taken the TOF BPM measurements keeping the buncher phase at -90 & BC on**
- ✓ **After this we requested Marsh to give the raw data file in his energy measurement program**

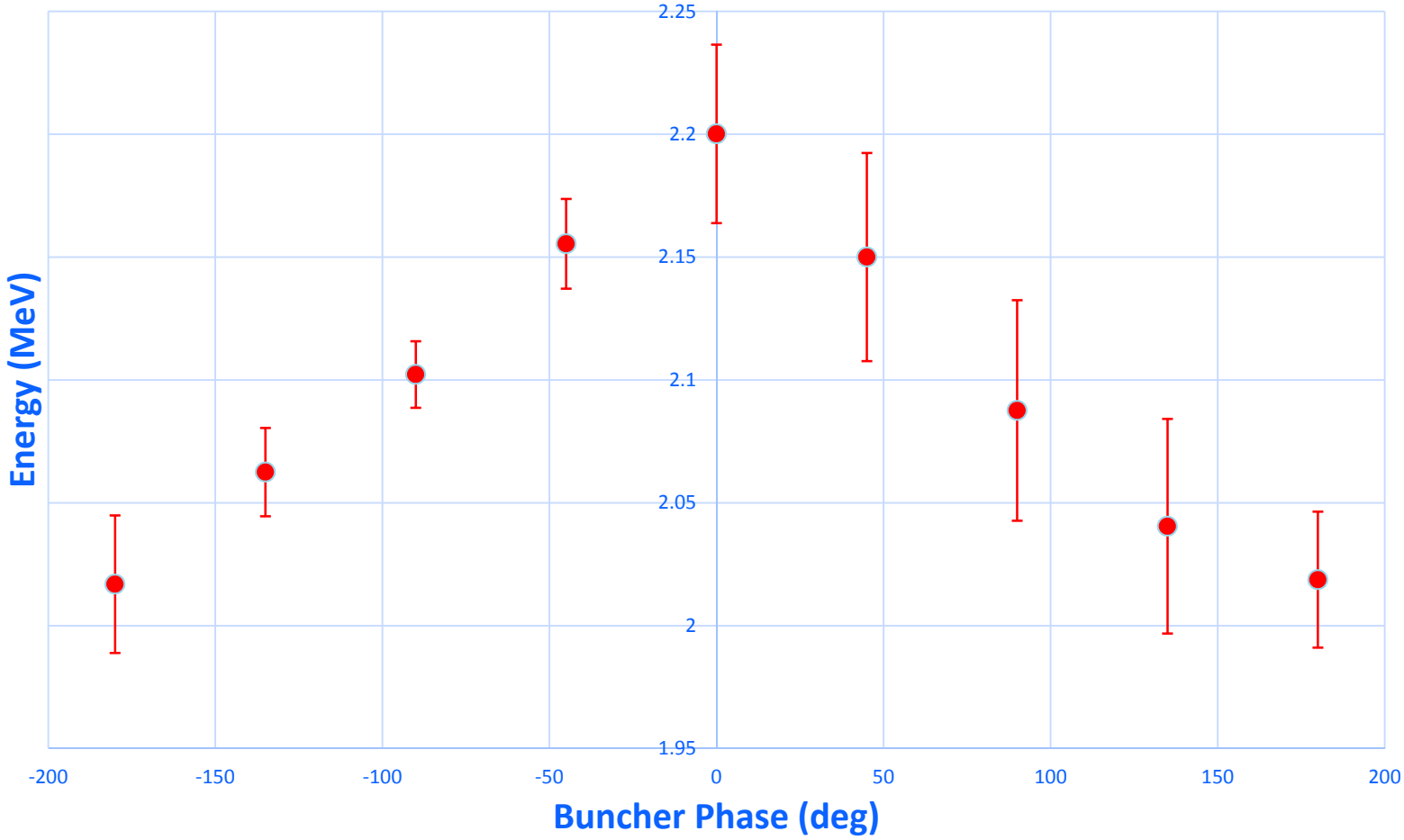
Energy of the beam using TOF BPM (30 averages & buncher at -90 deg)

Data taken by me and Jean-paul



The error in the energy measurements was within 1%

Energy error as a function of buncher phase (# of averages 30):



Energy error decreases when we are going towards bunching phase (-90 deg) and increases as we go away from -90 deg

Possible Errors in energy measurement:

- **Phase of the BPM**
- **Mechanical (motion of BPM)**

Slope error

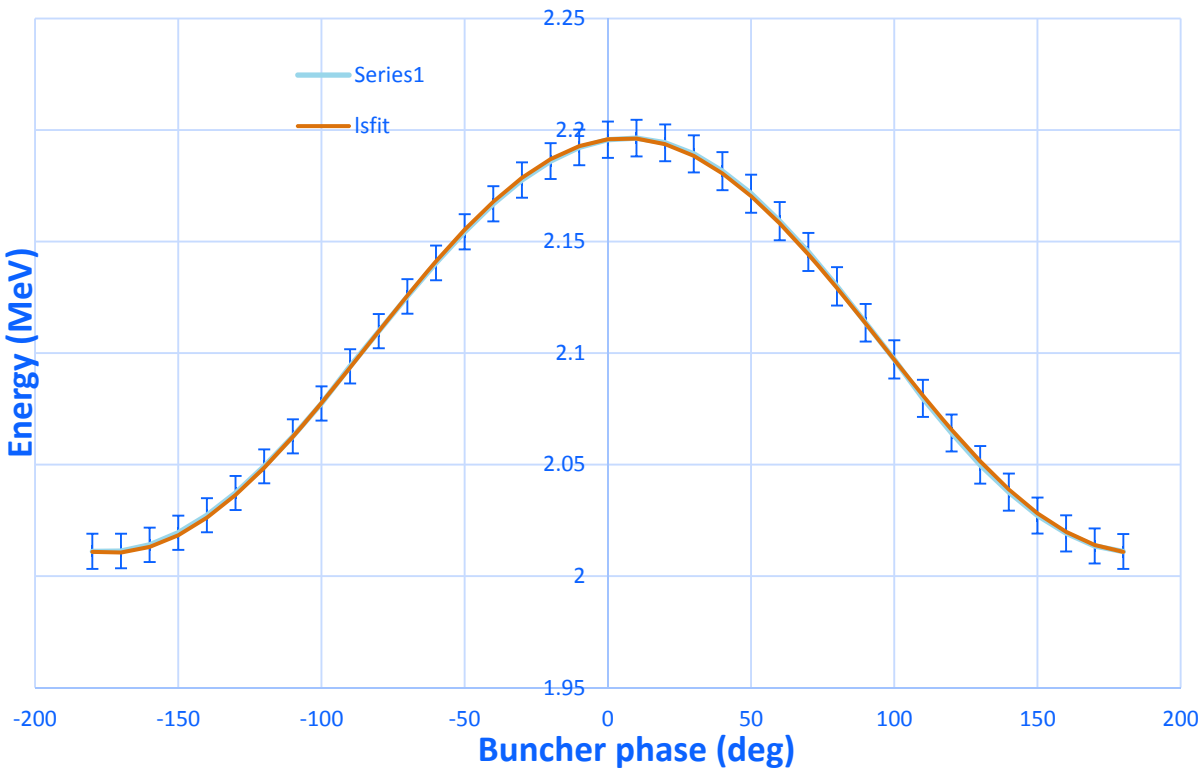
- **Resonant frequency offset (effect the accurate value of energy)**

$$E = \left[\frac{s}{\sqrt{s^2 - \left(\frac{0.36f}{300}\right)^2}} - 1 \right] m_0 c^2 \qquad dE = \sqrt{\left(\frac{\partial E}{\partial f}\triangle f\right)^2 + \left(\frac{\partial E}{\partial s}\triangle s\right)^2}$$

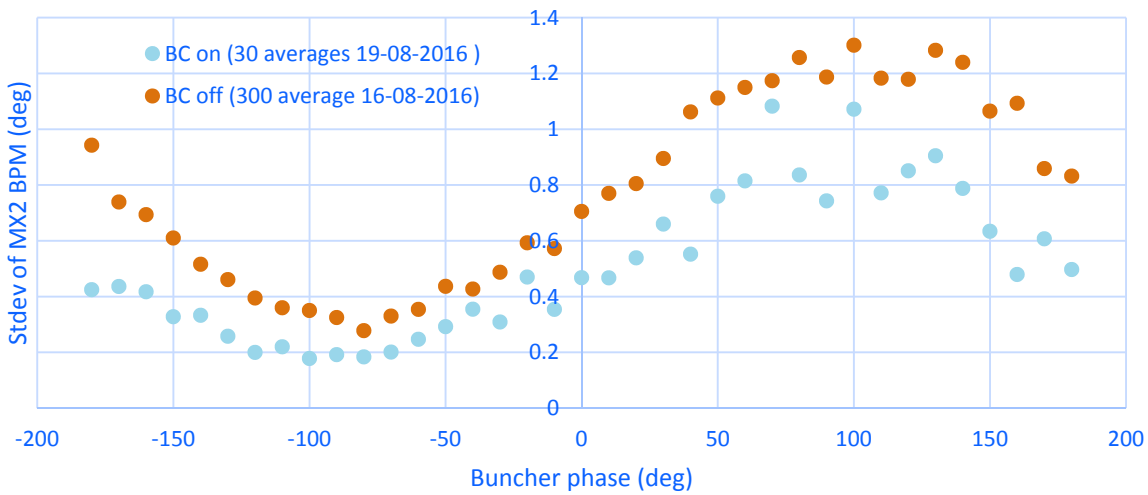
- Energy variation by an offset of resonant frequency by 30 kHz is 0.04%
- Energy error change by 0.05% for frequency error of 20 kHz
- So the slope error is the major contributor for the Energy error

Energy of beam using fixed BPMs:

- The energy of the beam is calculated by $\beta = \frac{s}{c\,(n\tau + \Delta t)}$, $\Delta t = \frac{\Delta\phi}{0.36 f}$
- The error in the velocity is given by $\sigma_\beta = \sqrt{\left(\frac{\partial\beta}{\partial s}\sigma_s\right)^2 + \left(\frac{\partial\beta}{\partial f}\sigma_f\right)^2 + \left(\frac{\partial\beta}{\partial\Delta t}\sigma_{\Delta t}\right)^2}$



- Energy errors are calculated using above formula.
- Assuming 20 kHz variation in f, and 1mm in length.
- We observed the σ in phase was minimum at around bunching phase
- Absolute value of energy depends on relative phases of BPMs.
- We have observed a relative phases of BPMs have changed



- It is clearly seen that stdev of BPMs reduce with BC on
- Errors in energy are 0.35% with BC on and averages of 30

Summary:

- Energy of the beam was measured with movable BPM method
  Errors are functions of buncher phase
  Minimum at -90 deg ~ 1% (major contribution from slope)
- Final RFQ energy is 2.11±1% (movable BPM), which is within the specs

- Energy measurement using two fixed BPMs are also done
  Errors are less when compared to movable BPM method
  But not so reliable for absolute energy measurement
  Because of change in relative phases of the BPMs
- Precesion was improved using BC in the two fixed BPMs method to around 0.4%

Thank you ...