1. Beam Stability in the Booster

Main challenges for the beam stability in the Booster are associated with transverse instabilities at injection and transition. Longitudinal instability at the transition as a potential source of the longitudinal emittance growth and some loss to the DC beam is considered elsewhere [46] and is not discussed below. Table 2.19 provides the main beam parameters.

Table 2.19: Beam in the Booster

Booster	Requirement	
Bunch population, N	$8.3\cdot 10^{10}$	
Longitudinal emittance, rms, $\varepsilon_{\parallel} = \sigma_\tau \sigma_E$	4.8	meV s
Maximal RF Voltage , V	0.75	MV
Maximal acceleration rate $\dot{\gamma}$	0.5	ms^{-1}
Transition gamma γ_t	5.47	

The Booster impedances are dominated by ones of the laminated magnets analytically found in Ref. [47,48]. Parameters of the focusing (F) and defocusing (D) Booster magnets are described in Sec. 2.3.4.

The transverse impedances of the focusing and defocusing magnets are shown in Figs. 2 and 3. The entire horizontal and vertical Booster impedances and wakes calculated with specified average betas at the magnet locations $(\beta_{xF}, \beta_{yF}, \beta_{xD}, \beta_{yD}) = (30\text{m}, 7\text{m}, 11\text{m}, 18\text{m})$ and the average beta $\bar{\beta} = 11\text{m}$ can be seen in Figs. 4 and 5.

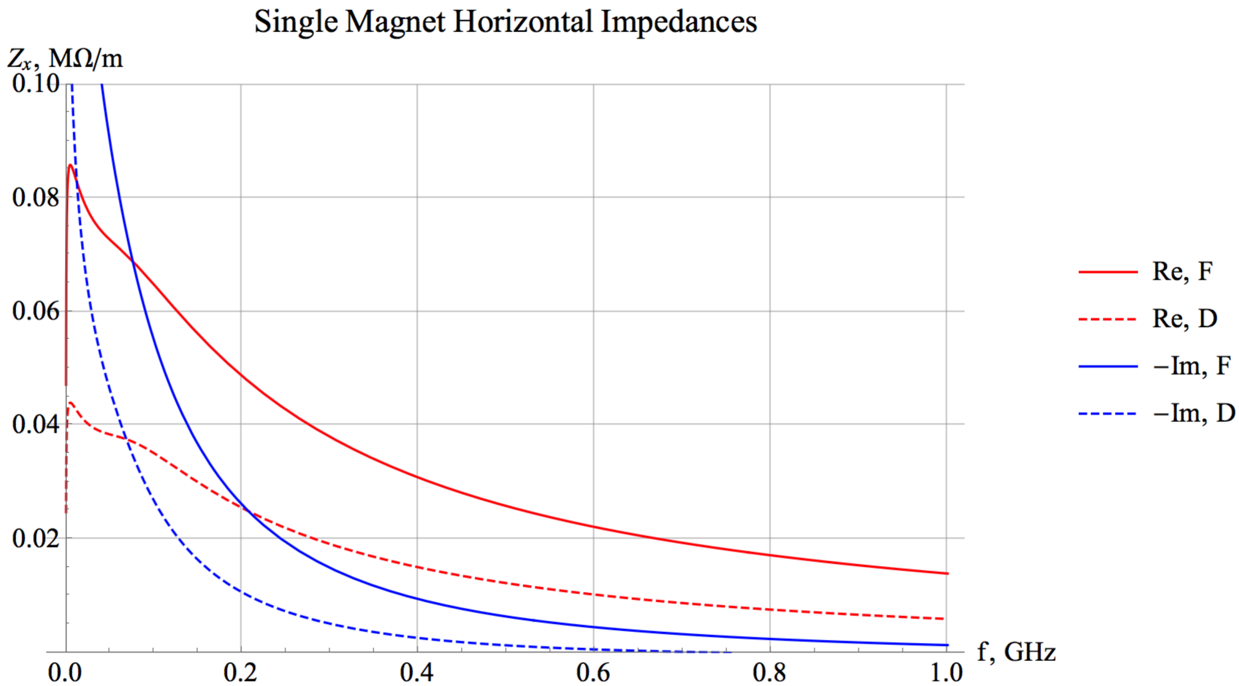

Fig. 2: Horizontal impedances of the focusing and defocusing magnets.

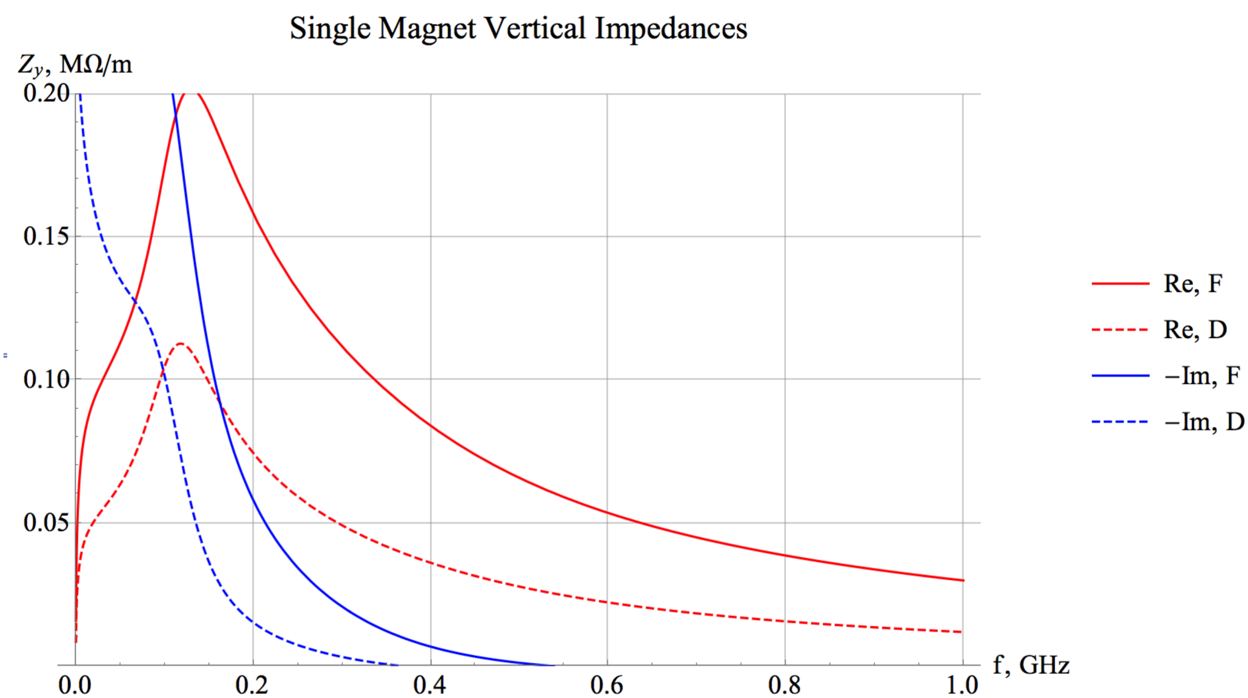

Fig. 3: Vertical impedances of the focusing and defocusing magnets.

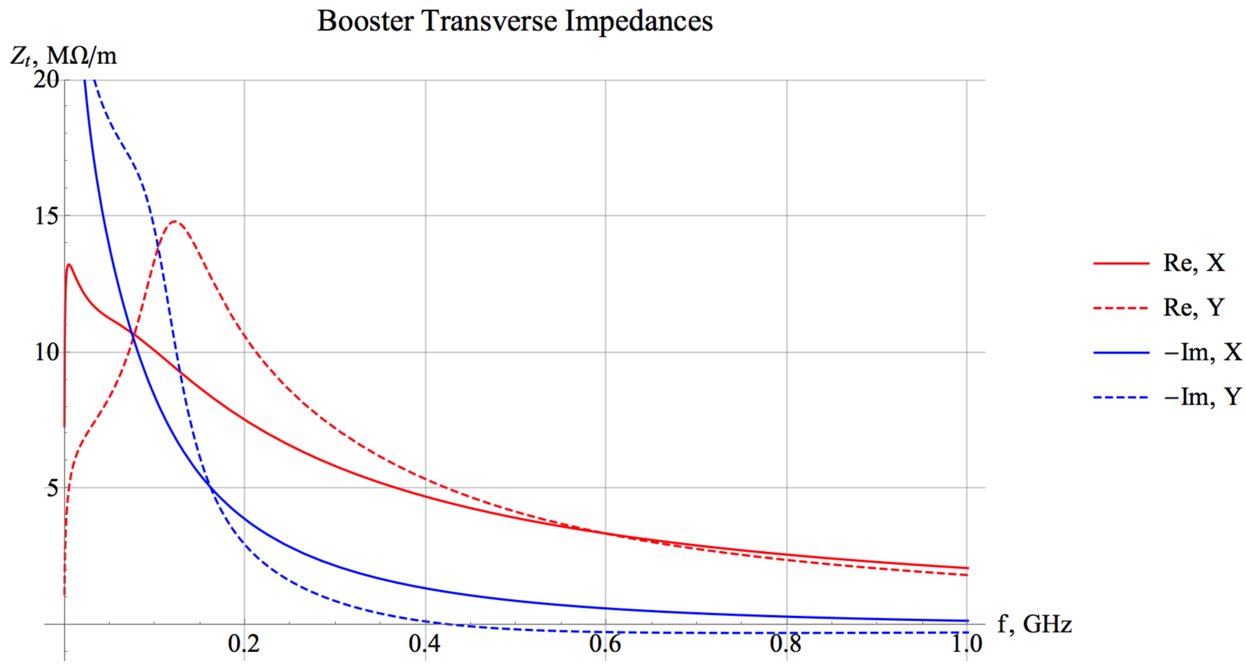

Fig. 4: Transverse impedances of the Booster.

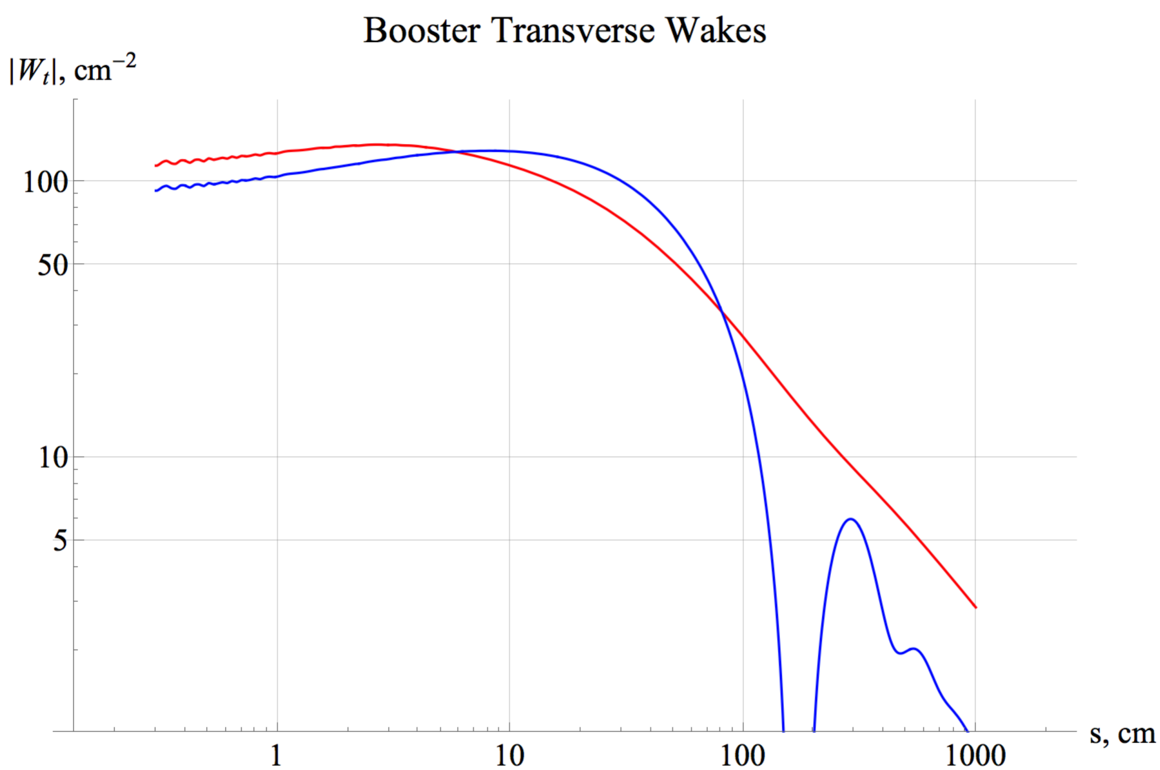

Fig. 5: Horizontal and vertical wakes of the Booster

For multi-bunch beams with strong space charge [49], the modes are characterized by two indexes: the single-bunch, (or head-tail, or intra-bunch) index n, and the coupled–bunch or inter-bunch index μ. At the first order of the perturbation theory over the wake function, the coherent tune shift $\Delta\nu_{n\mu}$ of the mode $|n,\mu\rangle$ is proportional to the sum of single- and coupled-bunch diagonal matrix elements of the wake function [49,50]:

$$
\begin{aligned}
&\Delta\nu_{n\mu} = -\frac{N r_0 \bar{\beta}\left(\hat{W}_n^{\mathrm{SB}} + \hat{W}_{n\mu}^{\mathrm{CB}}\right)}{4\pi\beta^2\gamma} \equiv \Delta\nu_{n\mu}^{\mathrm{SB}} + \Delta\nu_{n\mu}^{\mathrm{CB}};\\
&\hat{W}_n^{\mathrm{SB}} = \iint ds\, ds' \bar{W}(s-s')\exp(i\zeta(s-s'))\rho(s)\rho(s')y_n(s)y_n(s');\\
&\hat{W}_{n\mu}^{\mathrm{CB}} = W_\mu^{\mathrm{CB}}\left|I_n(\chi)\right|^2;\ I_n(\chi) = \int ds \exp(i\chi s/\sigma_s)\rho(s)y_n(s);\\
&\hat{W}_\mu^{\mathrm{CB}} = \sum_{k=1}^{\infty}\bar{W}(kC/M)\exp(ik\psi_\mu);\quad \psi_\mu = 2\pi\left(\{Q_b\}+\mu\right)/M;\quad \mu = 0,1,\ldots,M-1.
\end{aligned}
\tag{1.1}
$$

Here N is the number of particles per bunch, r_0 is the classical radius, β,γ are the relativistic factors, $\chi = \xi\sigma_s/(R\eta) \equiv \omega_\xi\sigma_\tau$ is the head-tail phase [Chao Eq. 6.187, p. 339 in Ref. [51]] with ξ as the chromaticity and η as the slippage factor, $\rho(s)$ is the normalized line density, $y_n(s)$ is the n-th head-tail eigen-function, M is the number of bunches and $\{Q_b\}$ is a fractional part of the betatron tune. At sufficiently small head-tail phase, the single-bunch growth rate can be neglected below TMCI threshold, while the coupled-bunch growth time is calculated as $\left[\omega_0 \operatorname{Im}\left(\Delta\nu_{00}^{\mathrm{CB}}\right)\right]^{-1} = 80\,\mu\mathrm{s}$ for the horizontal direction, in agreement with Ref [52]. A feedback can be taken into account similarly to the coupled-bunch wake [50]; for a bunch-by-bunch damper with a gain g this yields the damping rate

$$\operatorname{Im}(\Delta\nu_{n\mu}) = -g\left|I_n(\chi)\right|^2. \tag{1.2}$$

To make the description complete, the Landau damping has to be taken into account. To do that, the estimations of Landau damping suggested in Ref. [49] have to be compared with dedicated tracking simulations, e.g. with the Synergia [53]. When this is done and the full simulation scheme made complete, predictions and recommendations for various operation scenarios could be derived. An application of the bunch-by-bunch damper is known to be able to reduce the growth rate several times [BurovPRAB2016].

At the transition, the strong head-tail instability is suppressed by the chromaticity, which threshold value is proportional to the bunch population. Thus, increase of the latter by a factor of 1.5 compared with the current value would require similar growth of the former.

2. Beam Stability in the Recycler

Main parameters of the beam in the Recycler are listed in the Table below.

Table 2.1: Beam in the Recycler

Recycler	Requirement	
Bunch population, N	$8.0\cdot 10^{10}$	
Number of bunches	$81\cdot 6\cdot 2=972$	
Longitudinal emittance, rms, $\varepsilon_{\parallel}=\sigma_{\tau}\sigma_{E}$	5.5	meV s
Maximal RF Voltage , V	0.125	MV
Transition gamma γ_t	21.6	

At the injection with $\gamma=9.5$ the space charge is strong transversely, i.e. its tune shift ΔQ_{sc} is much higher than the synchrotron tune Q_s; the space charge is also important longitudinally, leading to about 20% of the synchrotron tune depression [51], and possibly to a longitudinal instability similar to the “dancing bunches” [54]. The longitudinal instability can be significantly enhanced by coupled-bunch interaction through high order modes (HOM) in the cavities, leading to the growth rate [55]:

$$\begin{gathered}\tau_{\parallel}^{-1}=\frac{MNr_0\eta R_s\omega_r\rho_{\omega_r}^2}{\gamma CQ_s Z_0};\\ \rho_{\omega_r}^2=\exp\left(-\omega_r^2\sigma_\tau^2\right);\quad Z_0=377\mathrm{Ohm}.\end{gathered}\tag{1.3}$$

Here R_s is the shunt impedance, ω_r is the HOM frequency, σ_τ is the rms bunch length in time units. For $R_s=35\mathrm{k\Omega},\ \omega_r/(2\pi)=150\mathrm{MHz},\ \sigma_\tau=1.9\mathrm{ns}$ this yields rather high frequency suppression factor $\rho_{\omega_r}^2=0.03$, leading to $\tau_{\parallel}=30\mathrm{ms}, \tau_{\parallel}^{-1}=0.02\omega_s$. If needed, a narrow-band damper could suppress such slow coupled-bunch motion.

Transverse single-bunch instability is described by a growth rate [49]

$$\tau_{\perp\mathrm{SB}}^{-1}=F_{\mathrm{SB}}(\chi)\frac{Nr_0\bar{W}_{\mathrm{SB}}\bar{\beta}}{4\pi\gamma}\ ,\tag{1.4}$$

where $F_{\mathrm{SB}}(\chi)\le 0.1$ is the chromaticity factor determined by the head-tail phase $\chi=\zeta\sigma_s$ $\bar{W}$ is a bunch-averaged wake function, $\bar{\beta}$ is the average beta-function. For $\chi\ge 1$, the chromaticity factor saturates at its maximum, $F_{\mathrm{SB}}(\chi)\simeq 0.1$. For the resistive wall case, with the half-gap b and the conductivity σ, the average wake is estimated as

$$\bar{W}_{\mathrm{SB}}=\frac{\pi^2}{12}\frac{2C}{\pi b^3\sqrt{\sigma\sigma_\tau}}\tag{1.5}$$

Altogether, this leads to single-bunch instability with a growth time $\tau_{\perp\mathrm{SB}}=20\mathrm{ms},\ \tau_{\perp\mathrm{SB}}^{-1}=0.03\omega_s$.

For a resistive wall coupled-bunch transverse impedance

$$Z_{\mathrm{CB}}=Z_0R\frac{\pi^2}{12}\frac{\delta}{b^3}\ ,\tag{1.6}$$

where δ is the skin depth at the corresponding coupled-bunch frequency, the growth rate can be represented as

$$\tau_{\perp \mathrm{CB}}^{-1} = \left| I_n(\chi) \right|^2 \frac{MNr_0\bar{\beta}}{\gamma T} \frac{Z_{\mathrm{CB}}}{Z_0 R} \tag{1.7}$$

with the coupled-bunch chromatic form-factor $I_n(\chi)$ given by Eq. (1.1). For the parameters of Table 2.22, this yields the growth rate close to the synchrotron frequency, $\tau_{\perp \mathrm{CB}}^{-1} = 1.2\omega_s = 1.8 \cdot 10^3 \mathrm{s}^{-1}$. Suppression of that fast instability most likely would require at least one of two means of suppression: either the transverse damper or nonlinear optical elements (octupoles or preferably e-lenses). Possible benefits of the transverse damper for the RR are discussed at Ref.[BurovCoupledBeam].

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