1. Beam Stability in the Booster

Main challenges for the beam stability in the Booster are associated with transverse instabilities at injection and transition. Longitudinal instability at the transition as a potential source of the longitudinal emittance growth and some loss to the DC beam is considered elsewhere [46] and is not discussed below. Table 2.19 provides the main beam parameters.

Table 2.19: Beam in the Booster

<table>
<thead>
<tr>
<th>Booster</th>
<th>Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bunch population, $N$</td>
<td>$8.3 \cdot 10^{10}$</td>
</tr>
<tr>
<td>Longitudinal emittance, rms, $\varepsilon_{|}=\sigma_{|}\sigma_{E}$</td>
<td>4.8 meV s</td>
</tr>
<tr>
<td>Maximal RF Voltage, $V$</td>
<td>0.75 MV</td>
</tr>
<tr>
<td>Maximal acceleration rate $\dot{\gamma}$</td>
<td>0.5 ms$^{-1}$</td>
</tr>
<tr>
<td>Transition gamma $\gamma_t$</td>
<td>5.47</td>
</tr>
</tbody>
</table>

The Booster impedances are dominated by ones of the laminated magnets analytically found in Ref. [47,48]. Parameters of the focusing (F) and defocusing (D) Booster magnets are described in Sec. 2.3.4.

The transverse impedances of the focusing and defocusing magnets are shown in Figs. 2 and 3. The entire horizontal and vertical Booster impedances and wakes calculated with specified average betas at the magnet locations $(\beta_{xF}, \beta_{yF}, \beta_{xF}, \beta_{yD}) = (30 \text{m}, 7 \text{m}, 11 \text{m}, 18 \text{m})$ and the average beta $\beta = 11 \text{m}$ can be seen in Figs. 4 and 5.
Fig. 2: Horizontal impedances of the focusing and defocusing magnets.

Fig. 3: Vertical impedances of the focusing and defocusing magnets.
Fig. 4: Transverse impedances of the Booster.

Fig. 5: Horizontal and vertical wakes of the Booster
For multi-bunch beams with strong space charge [49], the modes are characterized by two indexes: the single-bunch, (or head-tail, or intra-bunch) index $n$, and the coupled–bunch or inter-bunch index $\mu$. At the first order of the perturbation theory over the wake function, the coherent tune shift $\Delta v_{nj}$ of the mode $|n,\mu\rangle$ is proportional to the sum of single- and coupled-bunch diagonal matrix elements of the wake function [49,50]:

$$\Delta v_{nj} = -\frac{N r_0 \beta}{4 \pi \beta^2 \gamma} \left( \hat{W}_{n}^{SB} + \hat{W}_{n}^{CB} \right) \equiv \Delta v_{nj}^{SB} + \Delta v_{nj}^{CB},$$

$$\hat{W}_{n}^{SB} = \int ds ds' \tilde{W}(s-s') \exp(i \xi(s-s')) \rho(s) \rho(s') y_n(s) y_n(s');$$

$$\hat{W}_{n}^{CB} = W_{n}^{CB} |I_n(\chi)|^2; \quad I_n(\chi) = \int ds \exp(i \chi s / \sigma_s) \rho(s) y_n(s);$$

$$\hat{W}_{\mu}^{CB} = \sum_{k=1}^{\infty} \tilde{W}(kC / M) \exp(i k \psi_{\mu}); \quad \psi_{\mu} = 2 \pi \left( \{Q_n\} + \mu \right) / M; \quad \mu = 0, 1, \ldots, M - 1.$$  

Here $N$ is the number of particles per bunch, $r_0$ is the classical radius, $\beta, \gamma$ are the relativistic factors, $\chi = \xi \sigma_s / (RN) \equiv \omega_s \sigma_s$ is the head-tail phase [Chao Eq. 6.187, p. 339 in Ref. [51]] with $\xi$ as the chromaticity and $\eta$ as the slippage factor, $\rho(s)$ is the normalized line density, $y_n(s)$ is the $n$-th head-tail eigen-function, $M$ is the number of bunches and $\{Q_n\}$ is a fractional part of the betatron tune. At sufficiently small head-tail phase, the single-bunch growth rate can be neglected below TMCI threshold, while the coupled-bunch growth time is calculated as $\omega_0 \operatorname{Im}(\Delta v_{n0}^{CB})^{-1} = 80 \mu s$ for the horizontal direction, in agreement with Ref [52]. A feedback can be taken into account similarly to the coupled-bunch wake [50]; for a bunch-by-bunch damper with a gain $g$ this yields the damping rate

$$\operatorname{Im}(\Delta v_{nj}) = -g |I_n(\chi)|^2.$$  

To make the description complete, the Landau damping has to be taken into account. To do that, the estimations of Landau damping suggested in Ref. [49] have to be compared with dedicated tracking simulations, e.g. with the Synergia [53]. When this is done and the full simulation scheme made complete, predictions and recommendations for various operation scenarios could be derived. An application of the bunch-by-bunch damper is known to be able to reduce the growth rate several times [BurovPRAB2016].

At the transition, the strong head-tail instability is suppressed by the chromaticity, which threshold value is proportional to the bunch population. Thus, increase of the latter by a factor of 1.5 compared with the current value would require similar growth of the former.

### 2. Beam Stability in the Recycler

Main parameters of the beam in the Recycler are listed in the Table below.

#### Table 2.1: Beam in the Recycler
Recycler

<table>
<thead>
<tr>
<th>Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bunch population, $N$</td>
</tr>
<tr>
<td>Number of bunches</td>
</tr>
<tr>
<td>Longitudinal emittance, rms, $\varepsilon_| = \sigma_t \sigma_E$</td>
</tr>
<tr>
<td>Maximal RF Voltage, $V$</td>
</tr>
<tr>
<td>Transition gamma $\gamma_t$</td>
</tr>
</tbody>
</table>

At the injection with $\gamma = 9.5$ the space charge is strong transversely, i.e. its tune shift $\Delta Q_w$ is much higher than the synchrotron tune $Q_s$; the space charge is also important longitudinally, leading to about 20% of the synchrotron tune depression [51], and possibly to a longitudinal instability similar to the “dancing bunches” [54]. The longitudinal instability can be significantly enhanced by coupled-bunch interaction through high order modes (HOM) in the cavities, leading to the growth rate [55]:

$$\tau_\|^{-1} = \frac{MN_r \eta R_s \omega_0 \rho_0^2}{\gamma C Q Z_0};$$

$$\rho_0^2 = \exp(-\omega_0^2 \sigma_t^2); \quad Z_0 = 377\text{Ohm}.$$  (1.3)

Here $R_s$ is the shunt impedance, $\omega_0$ is the HOM frequency, $\sigma_t$ is the rms bunch length in time units. For $R_s = 35\text{k}\Omega$, $\omega_0 / (2\pi) = 150\text{MHz}$, $\sigma_t = 1.9\text{ns}$ this yields rather high frequency suppression factor $\rho_0^2 = 0.03$, leading to $\tau_\| = 30\text{ms}, \tau_\|^{-1} = 0.02\omega_0$. If needed, a narrow-band damper could suppress such slow coupled-bunch motion.

Transverse single-bunch instability is described by a growth rate [49]

$$\tau_\bot^{-1} = F_{s\bot}(\chi) \frac{N_r \bar{W}_{s\bot} \bar{\beta}}{4\pi \gamma},$$

(1.4)

where $F_{s\bot}(\chi) \leq 0.1$ is the chromaticity factor determined by the head-tail phase $\chi = \zeta \sigma_z$, $\bar{W}$ is a bunch-averaged wake function, $\bar{\beta}$ is the average beta-function. For $\chi \geq 1$, the chromaticity factor saturates at its maximum, $F_{s\bot}(\chi) = 0.1$. For the resistive wall case, with the half-gap $b$ and the conductivity $\sigma$, the average wake is estimated as

$$\bar{W}_{s\bot} = \frac{\pi^2}{12} \frac{2C}{\pi b^3 \sqrt{\sigma \sigma_z}}.$$  (1.5)

Altogether, this leads to single-bunch instability with a growth time $\tau_{s\bot} = 20\text{ms}, \tau_{s\bot}^{-1} = 0.03\omega_z$.

For a resistive wall coupled-bunch transverse impedance

$$Z_{CB} = Z_0 R \frac{\pi^2}{12} \frac{\delta}{b^5},$$  (1.6)
where \( \delta \) is the skin depth at the corresponding coupled-bunch frequency, the growth rate can be represented as

\[
\tau_{-1}^{-1} = |I_n(\chi)|^2 \frac{MNr_0 \bar{B} Z_{CB}}{\gamma T Z_0 R}
\]  

(1.7)

with the coupled-bunch chromatic form-factor \( I_n(\chi) \) given by Eq. (1.1). For the parameters of Table 2.22, this yields the growth rate close to the synchrotron frequency, \( \tau_{-1}^{-1} = 1.2 \omega_s = 1.8 \cdot 10^3 \text{s}^{-1} \). Suppression of that fast instability most likely would require at least one of two means of suppression: either the transverse damper or nonlinear optical elements (octupoles or preferably e-lenses). Possible benefits of the transverse damper for the RR are discussed at Ref.[BurovCoupledBeam].

References